Investigating the Mass Spectra of Di-Hadronic Molecule by Potential Model

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Abstract

In the present paper we address the enthralling problem of discovering and comprehending the hadronic-like molecular interactions. We use a Yukawa-like screening potential in s-wave state in conjunction with the One Boson Exchange potential to calculate the mass spectra. We suggest interaction between two hadrons, a dipole-like in colour-neutral state that results in the production of a hadronic molecule along with application of the compositeness theorem of Weinberg, which is used to differentiate these molecules from other hadronic states. We compute the mass spectra for various di-mesonic states, using the proposed interaction potential.

Keywords: Exotic; Hadronic-Molecule; Meson.
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Introduction

In recent decades, significant efforts have been directed toward the search for hadronic molecules, both theoretically and experimentally [1-7]. These molecules, akin to the deuteron which are strong candidates within the model of Quantum Chromodynamics (QCD).

QCD can comprehended as the fundamental theory describing strong interactions. Alongside the well-established conventional baryons (three-quark systems) and mesons (quark-antiquark pairs), hadronic molecules present an intriguing extension of QCD's predictive power.

Conventional baryons, such as protons and neutrons, serve as the important part of atomic nuclei, and their properties, including masses, magnetic moments, and decay modes, have been extensively studied using potential models, lattice QCD, and experimental data [8-16]. Similarly, mesons like Pions, kaons, and charmonium states (e.g., J/ψ and $\psi(2S)$) play critical roles in understanding hadronic interactions and mediating nuclear forces [17-18]. However, the discovery of exotic hadrons has extended the particle physics frontier beyond these conventional states. In recent years, several narrow exotic resonances have been observed, such as X(6900) [1]$P_c(4450)$[2], $Z_b(10610)$-/(10650) [4] X (3872) [5], Z $(4430)^+$ [6], Y (4260) [7], , and. These states, which defy straightforward classification within the conventional baryon-meson framework, have sparked intense interest in their underlying substructures.

Various theoretical explanation has been proposed for these exotic states, including:

- **Conventional heavy baryons**, [14-16] where exotic states are attributed to excited or unconventional configurations of three quark pairs with one or more heavy quarks.
- **Conventional heavy meson** [17-18]: where states are attributed to excited or unconventional configurations of quark-antiquark pairs with one or more heavy quarks.
- **Hadronic molecules** [19-20], which describe these states as loosely bound systems of two or more hadrons
- **Conventional light meson** [21]: where states are attributed to excited or unconventional configure-ions of quark-antiquark pairs with light quarks.
- **Conventional light baryons** [22-23]: where exotic states are attributed to excited or unconventional configurations of three light quark pairs.

- **Compact tetraquarks** [24-29], **Compact pentaquarks** [30] representing a tightly bound state of diquarks and antidiquarks.

The decay width (Gamma) in hadron spectroscopy signifies the stability of a particle, where a larger (Gamma) corresponds to a shorter lifetime. This classification aids in distinguishing decays into three categories: strong (rapid), electromagnetic (intermediate), and weak (gradual). Measuring (Gamma) plays a crucial role in the identification of exotic hadrons. Theoretical approaches to these models include effective field theory [31], QCD sum rules [3], lattice QCD [32], and potential models [33-34]. Comprehensive reviews, such as, consolidate these findings and provide an overview of their experimental and theoretical progress.

This study narrows its focus to the di-hadronic molecular model in a potential framework. primary questions drive this investigation are: (i) understanding how interactions between two colour-neutral hadrons lead to bound states, and (ii) differentiating hadronic molecular states from conventional hadrons or compact exotic states [27]. A key theoretical insight is that the One-Pion Exchange (OPE) potential provides sufficient long-range attraction to support molecular binding [28]. However, OPE alone fails to account for the very close-range interactions, necessitating inclusion for One Boson Exchange (OBE) potentials and Yukawa-like screening potentials [29]. Hadronic molecules provide good path to probe the strong force at quantum level. These systems, formed through delicate interactions between colour-neutral hadrons, highlight the intricate dynamics of QCD. Despite substantia l progress, many questions remain, particularly regarding the binding mechanisms, stability, and mass spectra of these states.

This study employs theoretical tools such as Weinberg's theorem to distinguish hadronic molecules from loosely coupled systems, providing a systematic approach to classification. Special emphasis is placed on calculating the mass spectra of di-hadronic states, which are significant for both theoretical exploration and experimental validation. By analysing their binding energies and stability, this work aims to uncover patterns inherent to hadronic molecular states. These findings not only deepen our understanding of strong interactions and QCD but also provide valuable insights for experimentalists working at facilities like PANDA, J-PARC, Belle, and LHCb [30]. As hadronic molecules bridge the gap between conventional baryons/mesons and exotic hadrons, further theoretical and experimental advancements are crucial for unravelling the full complexity of the strong force and its manifestations in nature. The current work is organized as follows: after the introduction in Section 1 theoretical, Section 2 describes the mass spectra generated by this formalism for hadronic molecule. Section 3 summarizes the results and Section 4 draws the final conclusion.

Theoretical Framework

Interhadronic Interaction

The deuteron, which is formed by the bound states of a P and N, is widely recognized as a molecule. To analyse and predict other hadronic compounds similar to the deuteron, it is helpful to use the deuteron itself as a model. Understanding such bound states involves addressing two fundamental questions: (i) why two color-neutral hadrons are attracted to one another and (ii) how 2 hadrons form a bound state. Regard to the definition of a molecule, a bound state requires a sufficiently strong attractive potential, which can be quantified by an effective coupling constant. Several realistic potentials, such as the Paris Group potential, the Nijmegen Group potential, and the full-Bonn (CD-Bonn) potential, have been developed to address the first question.

The potentials in question are derived from particle exchange, referred to as (OBE-P) Potentials, which extend Yukawa's pion exchange potential utilized to elucidate the nuclear force. The OBE-P encompasses long, medium, and short-range interactions via meson exchange, with the mass of the exchanged mesons dictating the force's range. Addressing the second inquiry why it is difficult for two color-neutral hadrons to establish a bound state—demands a nuanced comprehension of fundamental interactions at exceedingly short distances, where OBE-P exhibits limitations.

In order to distinguish the hadronic molecular states from other hadronic states, the Compositeness Theorem becomes essential. The challenge lies in identifying these molecular states accurately. To address this, we employ Weinberg's compositeness theorem, which provides a systematic framework for identifying molecular states by differentiating them from other hadronic configurations. This approach is critical for resolving the challenges faced in the hadronic molecular model and distinguishing these states from other potential forms of matter.

Effective Potential (One Boson Exchange + Yukawa - Screen like)

The di-hadronic molecule's Hamiltonian:

$$H = \sqrt{P^2 + m_{h_1}^2} + \sqrt{P^2 + m_{h_2}^2} + V_{hh} \tag{1}$$

Where H stands for the Hamiltonian or total energy of the system, P represents the momentum of the particles, and m_{h1} and m_{h2} are the masses of the first and second particles, respectively. V_{hh} is the potential term that describes interactions or external forces affecting the system.

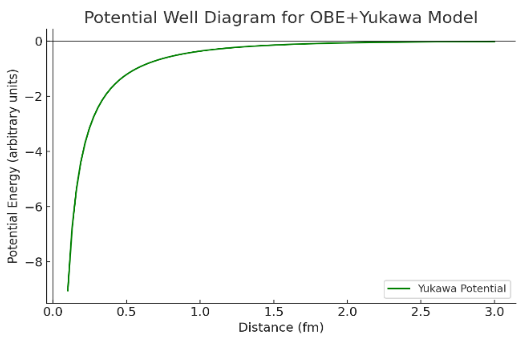

Figure 1: One Boson Exchange + Yukawa -Screen like potential

Through the utilization of the hydrogen-like trial wave function, we are able to ascertain the expected value of the Hamiltonian within the framework of the variational technique. We are able to determine the masses of low-lying di-mesonic states by using the expectation value of the Hamiltonian mathematical model.

$$H\psi = E\psi \quad \text{and} \tag{2}$$

$$\langle \text{Kinetic Energy} \rangle = \frac{1}{2}\left\langle r_{ij}\frac{dV_{hh}}{dr_{ij}}\right\rangle \tag{3}$$

and wave-function parameter can be obtain using virial theorem for each state.

The interaction potential, articulated by the One Boson Exchange potential (OBE-P) and a phenomenological attractive screened Yukawa-like potential [35] in the s-wave state, is elucidated in greater detail in the reference. The light mesons relevant to the One Boson Exchange potential (OBE-P) include π, η, σ, a0 (or δ), ω, and ρ. The OBE potential, which is the aggregate of all one-meson exchanges, namely

$$V_{OBE}=V_{ps}+V_s+V_v \tag{5}$$

Thus, Vps, Vs, and Vv represent the distinct s-wave one-meson exchange interaction potentials for pseudoscalar, scalar, and vector mesons, respectively. These potentials are articulated as

$$V_{ps} = \frac{1}{12}\left(\frac{g_{\pi qq}^2}{4\pi}\left(\frac{m_\pi}{m}\right)^2 \frac{e^{-m_\pi r_{ij}}}{r_{ij}}\right)\left[(\tau_i\cdot\tau_j) + \frac{g_{\eta qq}^2}{4\pi}\left(\frac{m_\eta}{m}\right)^2 \frac{e^{-m_\eta r_{ij}}}{r_{ij}}\right](\sigma_i\cdot\sigma_j) \tag{6}$$

Where Vps: Potential energy due to the pion and eta meson exchange interactions, $g_{\pi qq}$: Coupling constant for the pion-quark-quark interaction, m_π: Mass of the pion, m: Reference mass (e.g., the mass of the quark), r_{ij}: Distance between particles i and j, τi: Isospin operator for particles i and j, $g_{\eta qq}$: Coupling constant for the eta-quark-quark interaction. m_η: Mass of the eta meson, σi: Spin operator for particles i and j.

$$V_s = \frac{g_{\sigma qq}^2}{4\pi} m_\sigma \left[1 - \frac{1}{4}\left(\frac{m_\sigma}{m}\right)^2\right]\frac{e^{-m_\sigma r_{ij}}}{m_\sigma r_{ij}} + \frac{g_{\delta qq}^2}{4\pi} m_\delta \left[1 - \frac{1}{4}\left(\frac{m_\delta}{m}\right)^2\right]\frac{e^{-m_\delta r_{ij}}}{m_\delta r_{ij}}(\tau_i\cdot\tau_j) \tag{7}$$

Vs: Scalar potential energy due to the sigma and delta meson exchange interactions., $g_{\sigma qq}$: Coupling constant for the sigma-quark-quark interaction., m_σ: Mass of the sigma meson., $g_{\delta qq}$: Coupling constant for the delta-quark-quark interaction., m_δ: Mass of the delta meson., τ_{ij}: Isospin operator for particles i and j.

$$V_v = \frac{g_{\omega qq}^2}{4\pi}\left(\frac{e^{-m_\omega r_{ij}}}{r_{ij}}\right) + \frac{1}{6}\frac{g_{\rho qq}^2}{4\pi}\frac{1}{m^2}(\tau_i\cdot\tau_j)(\sigma_i\cdot\sigma_j)\left(\frac{e^{-m_\rho r_{ij}}}{r_{ij}}\right) \tag{8}$$

Vv: Vector potential energy due to the omega and rho meson exchange interactions, $g_{\omega qq}$: Coupling constant for the omega-quark-quark interaction, m_ω: Mass of the omega meson., $g_{\rho qq}$: Coupling constant for the rho-quark-quark interaction., m_ρ: Mass of the rho meson, τ·: Isospin operator for particles i and j, σi·σj: Spin operator for particles i and j.

The One Boson Exchange potential (OBE-P), incorporating finite size effects attributable to the extended structure of hadrons, can be articulated as

$$V_\alpha(r_{db}) = V_\alpha(m_\alpha, r_{db}) - F_{\alpha 2}V_\alpha(\Lambda_{\alpha 1}, r_{db}) + F_{\alpha 1}V_\alpha(\Lambda_{\alpha 2}, r_{db}) \tag{9}$$

Where Vα: Potential energy due to the interaction at distance r_{db}, Vα (mα, rdb): Potential energy term dependent on the parameter m_α and distance r_{db}, $F_{\alpha 2}$: Scaling factor for the term involving Λα1, Vα(Λα1,r_{db}):Potential energy term dependent on the parameter Λα1and distance r_{db}, Fα1:Scaling factor for the term involving Λα2., Vα(Λα2,r_{db}): Potential energy term dependent on the parameter Λα2 and distance r_{db}

Where α represents the η, σ, π, δ, ω, and ρ mesons, while

$$\Lambda\alpha 1=\Lambda\alpha+\epsilon \quad \text{and} \quad \Lambda\alpha 2=\Lambda\alpha-\epsilon$$

$$F_{\alpha 1} = \frac{\Lambda_{\alpha 1}^2 - m_\alpha^2}{\Lambda_{\alpha 2}^2 - \Lambda_{\alpha 1}^2} \quad \text{and}$$

$$F_{\alpha 2} = \frac{\Lambda_{\alpha 2}^2 - m_\alpha^2}{\Lambda_{\alpha 2}^2 - \Lambda_{\alpha 1}^2} \tag{10}$$

The subscript α refers to mesons such as ω, η, σ, π, δ, and ρ, where $(\epsilon/\Lambda_\alpha \ll 1)$, which suggests that a value of 10 MeV is a reasonable choice for the mass scale. To enhance the strength of the total effective dihadronic potential, a Yukawa screening potential is introduced [36]. This is necessary because the effective s-wave One Boson Exchange potential (OBE-P) is weak due to the highly sensitive cancellation between individual meson exchanges.

The residual running coupling constant, denoted as k_{mol}, and the screening fitting parameter (c) are introduced through the Yukawa-like potential. The intensity of the interaction can be modified using these parameters. A specific formula can estimate the residual running coupling constant (k_{mol}), incorporating the masses of the exchange mesons, the regularization parameter (Λ_α), and the coupling constants.

The estimation of the coupling constant is based on realistic potentials, which are modified to explain the properties of the deuteron using NN-phase shift data. For consistency, the same coupling constants used in the works of Machleidt (2001) and Weinberg (1965) for the deuteron are applied to other hadronic molecular systems. In this context, the meson-hadron coupling constants are approximated similarly to those used for nucleon-nucleon systems.

$$g_{\alpha hh} \cong g_{\alpha NN} \quad (11)$$

$g_{\alpha hh}$ and $g_{\alpha NN}$ represent the coupling constants for meson-hadron and meson-nucleon interactions, respectively.

The meson exchange's effective coupling constant exhibits scaling behaviour and has the ability to alter the interaction potential's strength. Knowing the coupling constant's effective strength allows us to obtain precise information about the degree of the hadron-meson-hadron interaction's individual meson exchange potential. In order to fix our model parameter, the meson-hadron coupling constant has been approximated.

The Yukawa-like screening potential was introduced to enhance the strength of the effective dihadronic interaction potential, as the efficacy of the effective s-wave one-boson exchange is constrained by the cancellation of individual meson exchanges. The Yukawa-type potential displayed is integrated.

$$V_Y = -\frac{k_{mol}}{r_{ij}} e^{-\frac{c^2 r_{ij}^2}{2}} \quad (12)$$

- The coupling constants are taken from realistic nucleon-nucleon potential models (Machleidt, 2001).
- Screening parameters are adjusted to fit deuteron binding energy for consistency.
- The meson masses are adopted from Particle Data Group (PDG) 2022.

V_Y: Potential energy due to the interaction, k_{mol}: A constant related to the molecular interaction strength, rij: Distance between particles i and j, c: A parameter related to the spread or range of the interaction.

In this context, k_{mol} signifies the running coupling constant, whereas c represents the screening fitting parameter. The value of k_{mol} can be calculated using the subsequent equation:

$$k_{mol}(M^2) = \frac{4\pi}{\left(11-\frac{2}{3}n_f\right)\ln\frac{M^2+M_B^2}{\Lambda_Q^2}} \quad (13)$$

Where $M = \frac{2md\ mb}{(md+mb)}$, md and mb are masses of constituent hadron which form di-hadronic molecule system. M_B = 950 MeV, Λ_Q is taken 399 MeV and 225 MeV for light and heavy meson, respectively. n_f is flavour number. The net inner hadronic potential V_{hh} is given below as

$$V_{hh} = V_{OBE} + V_Y \quad (14)$$

V_{OBE}: The overall binding energy is denoted by this term. It takes into consideration the energy needed to hold the components (such as quarks) of the hadron or molecule together. Understanding the stability and structure of the system depends on this binding energy.

V_Y: The interaction between particles mediated by mesons (such Pions or other mesons) is referred to as the Yukawa potential. Particularly in nuclear and particle physics, the Yukawa potential is crucial for comprehending the interaction between particles at a specific range.

The parameters that remain invariant in the current model encompass the masses, the exchange meson coupling constant, and Λ_Q, as referenced in sources. The computation of the residual running coupling constant, k_{mol}, is executed utilizing Equation (13). This model is adjusted to estimate the experimental binding energy of the deuteron by altering the color screening parameter, c, which acts as the sole free parameter.

It is important to note that three pseudoscalars at a single vertex do not conserve parity. Consequently, pseudoscalar exchanges between two pseudoscalar systems are not allowed by parity conservation. In the strong interaction, however, parity is a well-conserved quantum number. Hence σ, ω and a_0 exchanges donate to the One Boson Exchange (OBE) potential in the case $D^0\overline{K^{\pm}}$ and of $K\overline{K}$ and di-mesonic states. When calculating di-mesonic states, the σ -meson mass is taken as m_σ = 725 MeV for total isospin I_T=0, and m_σ= 500 MeV for I_T=1. For pseudoscalar-pseudoscalar states, such as f0(980) and a0(980), only σ, a_0 and ω exchanges are considered, as these exchanges are the sole contributors to the OBE potential of S-wave.

Results and Discussion

In the present study, the mass spectra of various dihadronic molecules have been calculated and analysed in compare-son with their corresponding threshold masses and experimental predictions. These results are summarized in Table 1, where the molecular states are categorized in accordance to their quantum numbers [37] JP and their natural energy scale. Calculated masses are benchmarked against meson thresholds to ensure accuracy. For this analysis, the parameters were adopted from the latest (Particle Data Group and PANDA) [38-44], ensuring consistency with current experimental and theoretical standards.

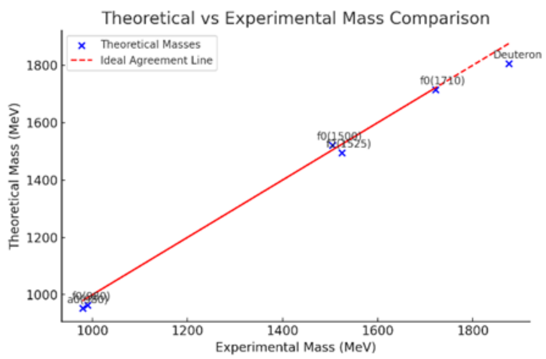

Figure 2: Theoretical vs Experimental Mass Comparison.

Conclusion and Future Prospective

In conclusion, the mass spectra of various di-hadronic molecules have been calculated within a non-relativistic framework. The methodology developed in this study lays the groundwork for future research aimed at exploring molecular hadrons composed of multiple hadrons. The findings from this work offer valuable insights and serve as a reference for experimental investigations conducted by future facilities such as PANDA, J-PARC, Belle, LHCb, PDG and others. This study adds to our growing understanding of unusual hadrons and their involvement in QCD. Further research in this subject will provide further insight into the underlying forces that regulate subatomic particles. The interaction of theory and experiment will remain critical in unravelling the riddles of hadronic molecules.

Table 1: The mass spectra of diverse molecular states, accompanied by their respective threshold masses in MeV.

Candidate	Molecular elucidation	S-Wave I(JP)	Threshold mass MeV	Exp.-mass MeV	This-Work mass MeV	Natural energy scale ($m_\pi^2/2\mu$) MeV	Uncertainty (MeV)
Deuteron	P n	0 (1+)	1877.84	1875.6	1805.5	19.91	±1.2
f0 (980)	K $\bar{K}$	0 (0+)	995.228	990 ± 2	964.07	37.02	±1.8
a0 (980)	K $\bar{K}$	1 (0+)	995.228	980 ± 20	953.17	37.03	±2.3
f0 (1500)	ρ ρ	0 (0+)	1550.98	1505 ± 6	1521.35	23.93	±3.1
f2(1525)	ρ ρ	0 (2+)	1550.98	1525 ± 5	1495.03	23.93	±3.5
f0(1710)	K* K*	0 (0+)	1791.8	1722±5	1715.22	22.21	±4.0

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