Nonlinearly Wave-Wave Interaction Leads to Solar Coronal Heating

B K Das[1,a], S K Singh[2], L K Gupta[3], R Prasad[4] and Priyank Kumar[5]

[1] Department of Physics, COER University, Roorkee-247667, Uttrakhand, India.
[2] Department of Physics, Govt College Lamta, Balaghat-481551, MP, India.
[3] Department of Physics, Shyamlal Saraswati Mahavidyalaya, Bulandshahr-203395, UP, India
[4] Department of Physics, Galgotia College of Engineering and Technology, Greater Noida-201310, UP, India.
[5] Department of Applied Sciences & Humanities, Government Polytechnic, Dahod-389151, Gujarat, India.

[a] drbkdas.iitd@gmail.com

Abstract

Nonlinear formulation of slow Alfvén wave in magnetized plasma which can be an essential ingredient of the solar space plasma is being studied. Various method has been adopted, mathematical formulation of the magnetohydrodynamic (MHD) waves can interpret the astrophysical phenomena happening in space plasma. The mathematical formulation of slow Alfvén wave and kinetic Alfvén wave (KAW) has been done from Maxwell equation as a make model equation. On the perturbation of slow Alfvén wave by pumped wave, the coupled wave dynamics has studied and their numerical simulation has been performed at $\theta = 50^0$. Several localized filamentary structures have observed with diverse intensities. The spectra associated with magnetic field fluctuations are also observed with Kolmogorov scaling for inertial and dispersive range spectral index which are proportional to $k^{-\frac{5}{3}}$ and k^{-3} respectively.

Keywords: Magneto Hydrodynamics (MHD), Solar Corona, Solar Space.
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*** Address of correspondence**
B K Das
Department of Physics, COER University,
Roorkee-247667, Uttrakhand, India.

Email: drbkdas.iitd@gmail.com

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Introduction

The general attribute of solar wind in space plasma is to eject constantly energetic particles in the space plasma. But what is actual mechanism behind the moving out of these highly energetic particles is still unanswered amid the group of investigators. Hence, it is even now an unlocked question in the area of astrophysics that how these energetic particles get accelerated [1]. The Alfvén waves (AWs) can be defined as an electromagnetic wave having low frequency travelling in magnetized plasma which take place due the equilibrium between the tension in magnetic field and ion inertia and therefore, they give the impression in the dynamics of the highly active particles in the space plasma [2]. Hence, it is assumed that AWs may be a good representative for transportation of momentum and energy in the various phenomena happening under geophysical and astrophysical scenario.

So many literatures expressed that the Alfvén waves are most imperative in laboratory plasma and space plasma. The various categories of these waves are able to explain how the transportation of energy takes place in the space plasma. It is, however, difficult to prove the detection of these waves. Recently, researchers [3-4] asserted the uncovering of these intangible Alfvén waves in various levels of solar atmosphere, for example spicules, sun umbra region and bright-points. In fact, these reports indirectly expressed about of Alfvén waves. Theoretically, considering the fully ionized plasmas they [5-6] have been thoroughly studied linear Alfvén waves. Many authors have used different methods to express damping process of moving Alfvén waves. For the damping process of linearity in the Alfvén have been studied by Leake et al. [7]. The heat released due to strongly collapsing of Alfvén waves have been explained by Song [8]. We have studied in a two-fluid model where the damping was occurred due to the collisions in the midst of electrons, ions, and neutrals, also due the collision of charged particles with EM field. Therefore, the researchers used this representation to study the chromosphere as well as the outcomes show that undamped part are closely associated with lower frequencies band which propagate via atmosphere, at the same time as higher frequencies band are intensely damped near to low altitudes. Under the consideration of Hall - term in the uniform and non-uniform plasmas, the dampness of Alfvén waves occur within the range of ion-cyclotron frequencies, have been studied by Threlfall et al. [9]. Lastly, Lazarian [10] explained the dampness of Alfvén waves is the effect of turbulence, which may have diverse applications in the area of astronomy and astrophysics. The Alfvén waves nonlinearity is important to investigate various nonlinear phenomena occurring in the space plasmas. It has been found that the linearly polarized characteristics of Alfvén wave are amplitude dependent, therefore, larger amplitudes these waves will have more density perturbations and also their motions in line with the magnetic field owing to ponderomotive force. Particularly, the equations of Alfvén wave in linearly polarized form and is travelling parallel ($||$) to magnetic field in a completely conducting fluid has been solved by Hollweg [11]. He found that the velocity and density fluctuations o longitudinal wave seem to be determined by the gradients of pressure exerted by magnetic field in the low-beta plasma regime. Rankin et al. [12] examined the nonlinearity associated with shear Alfvén waves travelling in the cold magnetized plasmas. They achieved analytical approximations in the limiting case β = 0. The filamentation of nonlinear waves gives us significant evidence for the dissipation problem, which leads to transfer momentum at lower scales. Some analytical studies have been done on the Alfvén waves (AWs)collapse. With small perturbation and interaction of Alfvén waves (AWs) [13] demonstrate that the configuration and development of lower-scale filamentary structures. Marsch et al., [14-15] have mathematically formulated and studied that Alfvén waves were exhibit in the solar wind travelling slowly. Parametric instabilities illustrated and predicted the non-linearity behaviour in the Alfvén waves [16]. The kinetic properties in parametric instability may play a key role which influences the ion dynamics. Consequently, they affect the instability growth rate [17-18]. So many authors [19-20] have already examined the nonlinear interaction of inertial AW with kinetic Alfvén wave. Tam and Chang [21] explained the intrigue of heating and acceleration mechanism of solar wind in which wave-particle interaction is the prime responsible for it. Zheng [22] advocated the special outcomes of nonlinear Alfvén wave. They found that the most important effect is dampness of Alfvén waves which in turn to the heating of plasma in one-dimensional systems but Alfvén wave builds up a damped soliton, when strong dissipation takes place.

Following kinetic theory method, Hasegawa and Chen [23] deliberated various nonlinear phenomena connected with behaviour of kinetic AWs. In situ observations, it has been revealed that variations in the frequency range from 10^{-4} Hz to 10^{-2} Hz. It lies between the state in which fast streams of charged particles deriving through coronal holes and Alfvén waves going away to the sun. This constant magnetic field amplitude as well as small density fluctuations, characterizes the Alfvénic state. The turbulence of slow solar wind gives an idea about the features that the wave's propagation is not easy to categorize and their density fluctuation spectra go along with Kolmogorov power law.

In this paper nonlinear formulation of slow Alfvénic wave have been derived and discussed. With the purpose of investigation the nonlinear property of waves, proper equation of slow Alfvén wave has been derived. Its significance is to explain the particles accelerating behaviour in solar wind as well as to interpret the mechanism of solar coronal heating.

Model Equations for Slow Alfvén Wave Dynamics

(i) Equation of particles flow:

$$\frac{\partial \vec{\upsilon}_j}{\partial t} = -\frac{q_j}{m_j}\vec{\nabla}\phi + \frac{q_j}{cm_j}\left(\vec{\upsilon}\times\vec{B}_0\right) - \frac{\gamma_j k_B T_j}{m_j}\vec{\nabla}\frac{n_j}{n_0} + \frac{\vec{F}_j}{m_j} \tag{1}$$

(ii) Navier-Stokes equation:

$$\frac{\partial n}{\partial t} + \vec{\nabla}.\left(n\vec{\upsilon}\right) = 0 \tag{2}$$

(iii) Maxwell-Faraday equation:

$$\left(\vec{\nabla}\times\vec{E}\right) = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \tag{3}$$

Where υ_j, m_j ,and T_j *j-type i, e*(ion and electron) c , n_0 and B_0 have usual meaning, velocity, masses, temperature, speed of light, density and B- field respectively and

$\vec{F}_j = \left[m_j\left(\vec{\upsilon}_j.\vec{\nabla}\right)\vec{\upsilon}_j - \left(\frac{q_j}{c}\right)\left(\vec{\upsilon}_j\times\vec{B}_0\right)\right]$ is defined for equation of the ponderomotive force. Also, Boltzmann relation expressed as $\phi = \left(\gamma_e k_B T_e/n_0 e\right)n_e - F_{ez}/eik_z$ where ω is ion acoustic and ω_{ci} is ion –cyclotron frequencies. The specific heat ratios γ_e and γ_i are for e=electron and i= ion. , k_B , T_e , n_e , ϕ and F_{ez} are defined for Boltzmann constant, electron temperature electron density, scalar potential and parallel component of electron pondermotive force respectively.

Where $C_s = \left(\frac{\gamma_e k_B T_e + \gamma_i k_B T_i}{m_i}\right)^{\frac{1}{2}}$ is the acoustic speed.

We consider that slow Alfvén wave is propagating of with frequency ω in the direction of *z*-axis, for which

$\vec{B}_0 = B_0\hat{z}$. Here, we consider *x-z* plane and it can be expressed as $\vec{k} = k_x\hat{x} + k_z\hat{z}$ and k is usually defined as wave number. Now, we can derive an equation for slow Alfvén wave from above well-known Maxwell's equations.

Operating with $\vec{\nabla}\times$ of Equation (3) and using Ampere's law we find

$$\nabla^2\vec{E} - \vec{\nabla}\left(\vec{\nabla}.\vec{E}\right) = \frac{4\pi}{c^2}\frac{\partial\vec{J}}{\partial t} + \frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} \quad (4)$$

The nonlinear current (NL) is defined as $\sim e(n_i\vec{v}_i - n_e\vec{v}_e) + en_0(\vec{v}_{i,NL} + \vec{v}_{e,NL})$.

We took the x-component of equation (4) and can be written as

$$\frac{\partial^2 E_x}{\partial z^2} - \frac{\partial^2 E_z}{\partial x\partial z} - \frac{1}{V_A^2}\frac{\partial^2 E_x}{\partial t^2} = \frac{4\pi n_0 e}{c^2}\frac{\partial(v_{ix}-v_{ex})}{\partial t} + \frac{4\pi n_0 e}{c^2}\frac{\partial(v_{iNL}-v_{eNL})}{\partial t} - \frac{1}{c^2}\frac{\partial^2 E_x}{\partial t^2} \quad (5)$$

We take the components *x*, *y* and *z* of $\vec{\upsilon}_j$ in equation (4), and hence the *x*-component can be written as:

$$\frac{\partial^2 E_x}{\partial z^2} - \frac{\partial^2 E_z}{\partial x\partial z} - \frac{1}{V_A^2}\frac{\partial^2 E_x}{\partial t^2} = -\beta\frac{m_e}{e}\omega^2\frac{\partial}{\partial x}\left(\frac{n_e}{n_0}\right) + \frac{4\pi ne}{c^2}\left[\frac{\omega^2 F_{ex}}{\omega_{ce}m_e} - \frac{i\omega F_{ey}}{\omega_{ce}m_e}\right] \quad (6)$$

Usually, the Alfvén wave speed is defined as $V_A = \left(B_0^2/4\pi n_0 m_i\right)^{1/2}$. Here, n_e, n_0, T_e have usual meanings. β is defined as $\beta = 8\pi N_0 T_e/B_0^2$. The approximate relationships between electric field components for each other are as follows (nonlinear terms can be neglected): $E_y = \frac{-iD}{(S-\eta^2)}E_x$, $E_z = \frac{\eta^2\cos\theta\sin\theta}{\left(P-\eta^2\sin^2\theta\right)}E_x$

where

$\eta = \frac{k^2c^2}{\omega^2}$, $S = 1 - \sum_j \frac{\omega_{pj}^2}{(\omega_0^2-\omega_{cj}^2)}$, $D = \sum_j \omega_{cj}\times\frac{\omega_{pj}^2}{(\omega_0^2-\omega_{cj}^2)}$ and $P = 1 - \sum_j\left(\frac{\omega_{pj}^2}{k_z^2V_{Tj}^2} - \frac{\omega_{pj}^2}{\omega^2}\right)$. Here, we have taken relationship derived by Stix [24]. Therefore, continuity equation expressed as

$$\frac{\partial}{\partial t}\left(\frac{n_e}{n_0}\right) = -\frac{\partial}{\partial x}(v_{ex}) - \frac{\partial}{\partial z}(v_{ez}) \quad (7)$$

From the pondermotive force we have

$$F_{jy} = \frac{q_j}{cB_0}\frac{\partial}{\partial x}\left\langle\left|E_z\right|^2\right\rangle + \frac{q_j^2}{m_j\omega^2}\left[E_z^*\frac{\partial}{\partial z}E_x\right] \quad (8)$$

and

$$F_{jy} = \frac{iq_j}{cB_0}\left[E_z^*\frac{\partial}{\partial z}E_x\right] \quad (9)$$

Now, we put the E_x, E_y in equation (4) and using pondermotive force components, one can find

$$\left[\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{V_A^2}\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial^2}{\partial z^2} - \frac{1}{V_A^2}\frac{\partial^2}{\partial t^2}\right) + \frac{\beta}{V_A^2}\left(\frac{\omega_{pe}^2}{\omega_{ce}^2} - \frac{c^2}{V_{Te}^2}\right)\frac{\partial^4}{\partial x^2\partial t^2}\right]\left(\frac{n_e}{n_0}\right) = \frac{2\omega_{pe}^2}{\omega_{ce}^2B_0^2}\frac{\partial}{\partial x}\left[\left(\frac{\partial B_{0y}}{\partial x}\right)\left(\frac{\partial^2 B_{0y}}{\partial z^2}\right)\right] - \frac{c^2}{4\omega_{ce}^2B_0^2V_A^2}\frac{\partial^4}{\partial x^2\partial t^2}\left|\frac{\partial B_{0y}}{\partial x}\right|^2 \quad (10)$$

For $\beta = 0$, we can recover slow and fast modes of the cold plasma. After Fourier transform LHS of equation (10) and equated RHS with zero, we get,

$$\left[\left(k^2 - \frac{\omega^2}{V_A^2}\right)\left(k_\parallel^2 - \frac{\omega^2}{V_A^2}\right) + \beta k_x^2\left(\frac{\omega_{pe}^2}{\omega_{ce}^2}\frac{\omega^2}{V_A^2} - \frac{c^2}{V_{Te}^2}\frac{\omega^2}{V_A^2}\right)\right]\left(\frac{n_e}{n_0}\right) = 0 \quad (11)$$

Therefore, $k_\parallel$ (parallel component) gives the expression for slow Alfvén wave which is propagating with low frequency and in dimensionless form of slow Alfven equation is obtained as

$$\left[\frac{\partial^2 n_e}{\partial z^2} - \zeta_1\frac{\partial^2 n_e}{\partial t^2}\right] = -\left|B_{0y}\right|^2 - \zeta_2\frac{\partial^2}{\partial z^2}\left|B_{0y}\right|^2 \quad (12)$$

$$\left[\frac{\partial^2}{\partial z^2} - \zeta_1\frac{\partial^2}{\partial t^2}\right]n_e = -\left|B_{0y}\right|^2 - \zeta_2\frac{\partial^2}{\partial x^2}\left|B_{0y}\right|^2 \quad (13)$$

Where $\zeta_1 = \frac{\omega^2}{k_{0x}^2V_A^2}\frac{(k_{0x}\lambda_e)^4}{\left(1+k_{0x}^2\lambda_e^2\right)^2}$ *and* $\zeta_2 = \frac{c^2\omega^2k_{0x}^2}{2\omega_{pe}^2k_{0z}^2V_A^2}$

The equation (13) is our desired nonlinear slow Alfvén wave equation responsible for acceleration of charged particles and solar coronal heating effect. For the calculation of constants, some important parameters nearly 1AU for solar wind plasma is taken [25].

Kinetic Alfvén Wave (KAW)

The nonlinear KAW is travelling along z-axis. The magnetic field B_0 associated with it. The equation is derived by [26-28]

$$\frac{\partial^2\tilde{B}_y}{\partial t^2} = \Gamma_1\lambda_e^2\frac{\partial^4\tilde{B}_y}{\partial x^2\partial t^2} - \Gamma_2V_{Te}^2\lambda_e^2\frac{\partial^4\tilde{B}_y}{\partial x^2\partial z^2} + V_A^2\left(1-\frac{n_e}{n_0}\right)\frac{\partial^2\tilde{B}_y}{\partial z^2}, \quad (14)$$

For low-β plasmas ($\beta \ll m_e/m_i$), $\Gamma_1 = 1$ and $\Gamma_2 = 0$

For intermediate-β plasmas ($\frac{m_e}{m_i} \ll \beta \ll 1$), $\Gamma_1 = 0$ and $\Gamma_2 = 1$ Electron skin depth, $\lambda_e = \left(\sqrt{c^2m_e/4\pi n_0e^2}\right)$

Suppose a solution for (14) as

$$\tilde{B}_y = B_{0y}\left(x,z\right)e^{i\left(k_{0x}x+k_{0z}z-\omega t\right)}. \quad (15)$$

From equation (14) and (15), we get

$$-2i\omega\frac{\partial B_y}{\partial t}-\left(2ik_{0z}k_{ox}^2\lambda_e^2V_{Te}^2\right)\frac{\partial B_y}{\partial z}-\left(k_{0x}^2V_{Te}^2\lambda_e^2\right)\frac{\partial^2 B_y}{\partial z^2}-\left(V_{Te}^2\lambda_e^2k_{0z}^2\right)\frac{\partial^2 B_y}{\partial x^2}$$
$$-2ik_{0x}\lambda_e^2\left(V_{Te}^2k_{0z}^2\right)\frac{\partial B_y}{\partial x}-k_{0z}^2V_A^2\left(1-n_e/n_0\right)B_y=0. \quad (16)$$

The system of equations namely (13) and (15), we can write in the dimensionless form as:

$$\left[\frac{\partial^2 n_e}{\partial z^2}-\zeta_1\frac{\partial^2 n_e}{\partial t^2}\right]=-\left|B_{0y}\right|^2-\zeta_2\frac{\partial^2}{\partial x^2}\left|B_{0y}\right|^2 \text{ and}$$

$$+i\frac{\partial B_y}{\partial t}-i\frac{\partial B_y}{\partial x}+i\frac{\partial B_y}{\partial z}+\zeta_3\frac{\partial^2 B_y}{\partial z^2}+n_eB_y=0, \quad (17)$$

$B_0=6.2\times10^{-5}G$, $c_s=4.63\times10^6$ cms^{-1}, $n_0=8.7cm^{-3}$, $T_e=1.4\times10^5K$, $T_i=1.2\times10^5K$, $V_A=4.6\times10^6$ cms^{-1}, $V_{th,e}=1.457\times10^8$ cms^{-1}, $\omega_{ci}=0.6$ Hz, $\omega_{pe}=5.64\times10^4$ Hz and $\beta\approx2\times10^{-4}$

For $\omega=0.1$ Hz and $\lambda_e=5.32\times10^3$ cm, we find $\beta=0.857$, $k_{0x}=6.5\times10^{-8}cm^{-1}$, $k_{0z}=5.83\times10^{-8}cm^{-1}$.

From the above parameters, the value of ζ_1 is very less as compared to ζ_2 therefore, it can be neglected.

Simulations

The equations (13) and (15) in a $\left(\frac{2\pi}{\alpha_x}\right)\times\left(\frac{2\pi}{\alpha_z}\right)$. Suppose a spatial domain with the wave numbers α_x and α_z to be periodic. (let their perturbation α_x and α_z equal to 0.2, has been solved numerically. Hare, $(256)^2$ grid points are taken in the spatial domain. Here we use a numerical method namely, Pseudo spectral approach for finite difference as well as Predictor-Corrector for space integration. Let there be the evolution in time with $dt=5\times10^{-5}$.

The equations involved in the numerical simulations are

$B_y\left(x,z,0\right)=B_{y0}\left(1+0.1\cos(\alpha_x x)\right)\left(1+0.1\cos(\alpha_x z)\right)$ and $n\left(x,z,0\right)=\left|B_y\left(x,z,0\right)\right|^2$

Initially, we take B_{y0}=1 for pump slow Alfvén wave amplitude. The Fig.1, characterizes the strength of localized B- field structures. Consequently, the coupling of waves, affects the dynamics. As a result, several filamentary structures are appeared at $\theta=50^0$ by maximum intensity $\left|B_y\left(x,z\right)\right|^2\approx10$, has been observed. The Figure 2, shows fluctuations power spectra in the graph of $|B_k|^2$ Vs k, which follows Kolmogorov scaling for inertial as well as dispersive range i.e. $k^{-5/3}$ and k^{-3} respectively.

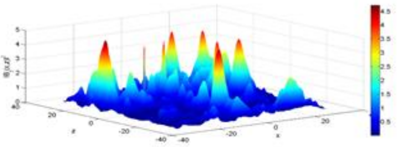

Figure 1: 3D-Filamentary structure of magnetic field strength for Slow Alfven wave at $\theta=50^0$

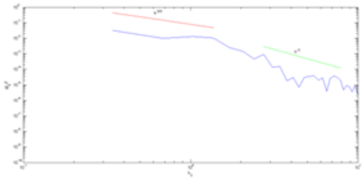

Figure 2: Evolution of magnetic strength fluctuations power spectra at $\theta=50^0$ at the end of the simulation

Discussion and Conclusion

This section of the paper summarizes formulation of the nonlinear slow of Alfvén Wave which is an important equation for the calculation of energy dissipation. On the perturbation of slow Alfvén wave by pumped wave, the coupled wave dynamics has studied and their numerical simulation has been performed at $\theta=50^0$. Several localized (x,z) filamentary structures have observed with diverse intensities. The spectra associated with magnetic field fluctuations are also observed with Kolmogorov scaling for inertial and dispersive range spectral index which are proportional to $k^{-5/3}$ and k^{-3} respectively.

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