Primordial Black Holes as Dark Matter Candidates in a Cyclic Universe

Bijan Kumar Gangopadhyay

Abstract

This paper explores the role of primordial black holes (PBHs) as dark matter candidates within a cyclic universe framework. The model employs a scalar field to drive expansion, contraction, and bounce cycles, with PBHs persisting as stable dark matter components. Our analysis of PBH density evolution suggests that their interactions with the scalar field and visible matter contribute to mass-energy continuity across cycles. Numerical simulations reveal that PBHs account for approximately 2.6% of the total dark matter density. Additionally, our model predicts that supermassive black holes (SMBHs) gradually lose mass due to Hawking radiation and dark matter interactions, affecting cosmic structure and evolution. These findings underscore the potential role of PBHs in cyclic cosmology and dark matter composition.

Keywords: Cyclic Universe, Primordial Black hole, Dark matter, SMBH.
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Introduction

The cyclic universe model proposes that the universe undergoes periodic cycles of contraction and expansion, with each new cycle emerging from the collapse of the previous one. This framework offers an alternative to traditional cosmological models by addressing challenges such as the initial singularity and entropy growth [1-2]. A central dogma of the cyclic universe is the role of dark matter, potentially in the form of primordial black holes (PBHs), and dark energy in driving these cycles. During the contraction phase, supermassive black holes (SMBHs) form by accreting both baryonic and dark matter. These SMBHs undergo gradual evaporation through Hawking radiation [3], setting the stage for a subsequent expansion phase. Crucially, dark matter, particularly PBHs, persists as a relic across cycles, influencing the dark matter content of each new universe. This study builds on previous work in cyclic cosmologies [1] by integrating PBHs as the most viable dark matter candidates and investigating their implications for the formation and evolution of SMBHs within a contracting universe.

Primordial black holes are expected dark matter candidates because they fulfill all the necessary criteria: they are cold, non-baryonic, stable, and can form in the appropriate abundance [4-5]. The theoretical groundwork for PBH formation was laid by Zel'dovich and Novikov (1967) [6], who proposed that PBHs could form from over densities in the early universe. This idea was further developed by Carr and Hawking (1974) [7], who explored the conditions under which PBHs might arise in the early cosmic environment. Since primordial black holes (PBHs) form before nucleosynthesis, they are inherently non-baryonic in nature, as their formation predates the production of light elements and baryonic matter during Big Bang Nucleosynthesis (BBN) [7-8]. Their non-baryonic nature makes them viable candidates for dark matter, as they remain decoupled from baryonic processes in the early universe [9-10]. Additionally, PBH formation theories suggest that they can emerge from a variety of mechanisms, including density fluctuations from inflation, phase transitions, or the collapse of cosmic strings [8-9,11].

Recent detection of gravitational waves mergers of tens-of-solar-mass black hole binaries has led to a surge in interest in Primordial Black Holes (PBHs) as a dark matter candidate [4,12-13]. These observations, combined with theoretical advancements [10], suggest that PBHs could play a dual role in cosmology: as contributors to the dark matter content and as essential elements in preserving matter across successive cosmic cycles. By integrating PBHs into the framework of cyclic cosmologies, this work provides new insights into the interaction between dark matter, dark energy, and the large-scale dynamics of the universe. Our model investigates the formation and evolution of SMBHs within a contracting universe and examines how dark matter, alongside dark energy, influences these dynamics. PBHs, which arise due to early-universe density fluctuations, play a crucial role in preserving dark matter across cosmic cycles [10].

Mathematical Framework and Numerical Analysis of The Universe Evolution:

The mathematical formulation underlying this work is based on the framework of General Relativity, where the dynamics of the universe are governed by the Friedmann equations. In this model, the universe is assumed to be homogeneous and isotropic, described by Friedmann-Lemaître-Robertson-Walker (FLRW) metric. The contributions from dark matter, baryonic matter, dark energy, and a scalar field driving cyclic evolution [14] are all taken into account. The cyclic nature of the universe, as proposed in this work, involves successive periods of expansion followed by contraction, culminating in a bounce, which initiates a new expansion phase. A scalar field, ϕ, with a potential V(ϕ) plays a crucial role in governing the transition between these phases.

Friedmann Equations and Energy Densities:

The Friedmann equations, derived from Einstein field equations, describe the evolution of the scale factor a(t), which defines the size of the universe as a function of time. For a flat universe, the simplified form of the Friedmann equation is given by:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left(\rho_\phi + \rho_{\text{matter}} + \rho_{DE}\right) \qquad (1)$$

where $H = \dot{a}/a$ is the Hubble parameter, ρ_ϕ is the scalar field energy density, ρ_{matter} includes contributions from baryonic and dark matter, and ρ_{DE} represents the dark energy density. The individual components of the energy densities are expressed as: $\rho_{\text{matter}} = \frac{\Omega_b+\Omega_{DM}}{a^3}$, where Ω_b is the baryonic matter density parameter and Ω_{DM} is the dark matter density parameter. $\rho_{DE} = \rho_{DE_0}$, representing a constant dark energy density. $\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$, where ϕ is the scalar field, $\dot{\phi}$ its time derivative, and $V(\phi)$ its potential energy. The evolution of the scalar field ϕ is represented by the Klein-Gordon equation [15-16].

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0, \qquad (2)$$

where $\ddot{\phi}$ is the second derivative of the scalar field, and the term $3H\dot{\phi}$ acts as a frictional term, representing the expansion of the universe.

The Scalar Field Potential and Bounce Mechanism:

In this model, the scalar field is responsible for driving the cyclic evolution of the universe, particularly the transition between expansion, contraction, and the bounce. During the contraction phase, the scalar field's potential takes on a negative quartic form as we propose:

$$V(\phi) = -\frac{1}{4}\phi^4 \qquad (3)$$

The necessity of a negative quartic scalar field potential arises from its role in facilitating a smooth contraction phase, ensuring the stability of the universe before the bounce. The negative potential introduces a repulsive effect that counteracts premature collapse, stabilizes scalar field energy, and aligns with cyclic cosmology principles. This approach extends ekpyrotic cosmology, where steep negative potentials suppress anisotropies and regulate contraction. Further exploration of its theoretical origin will strengthen its validity within the model.

1. **Negative pressure contribution:** The term naturally provides a source of negative pressure which aids in countering any premature collapse or instability during contraction.
2. **Scalar field evolution:** The quartic form offers a steep yet smooth potential that facilitates controlled scalar field dynamics, ensuring that the energy density evolves appropriately as the universe contracts.
3. **Compatibility with Cyclic Dynamics:** The potential aligns with the requirement of cyclic models, where a carefully tuned scalar field potential is often necessary to regulate the energy conditions and ensure a consistent periodic behavior. While this potential has not been directly derived from first principles or symmetry considerations, its form is phenomenologically consistent with scalar field theories in cyclic or ekpyrotic cosmology [17]. The proposed scalar field potential during the contraction phase shares certain similarities with the ekpyrotic framework, where in steep or negative potentials are employed to regulate the driving dynamics and to suppress instabilities as we desire. Ekpyrotic cosmology, as introduced in the context of brane-world scenarios, has demonstrated how scalar field dynamics can drive smooth contraction phases while addressing cosmological problems such as anisotropies [1, 17]. Building upon these ideas, our model extends this approach by introducing a negative quartic potential, which provides a novel mechanism for achieving the desired contraction behavior. Importantly, we find that introduction of this potential leads to key physical outcomes, including stabilization of scalar field energy during contraction, a smooth transition to the bounce phase without producing high curvature or instability, and the compatibility with observed cosmological constraints, particularly regarding scalar field evolution in high-density regimes. This proposal represents a novel approach to addressing the scalar field dynamics during contraction. While further theoretical search into the origin and deeper insight of the potential $V(\phi) =$

$-\frac{1}{4}\phi^4$ is warranted its inclusion in our model offers a robust framework for understanding the contraction phase in the cyclic universe. This negative quartic potential plays a crucial role in initiating the contraction of the universe, leading to the bounce. The bounce occurs when the scale factor a(t) reaches a minimum, marking a transition point where contraction halts and the universe begins to re-expand.

Numerical Evolution and Initial Conditions:

To explore the dynamics of this cyclic universe model, we perform numerical simulations of the evolution of the scale factor and the scalar field. The initial conditions are chosen as follows:

Initial Hubble parameter: $H_0 = 1$,
Initial scalar field value: $\phi_0 = 0.1$,
Initial derivative of scalar field: $\dot{\phi}_0 = 0$,
Initial scale factor: $a_0 = 0.1$.

These initial conditions allow us to simulate the behaviour of the universe as it contracts, reaches the bounce, and then oscillates. This subsection analyses the behaviour of the universe as it evolves toward contraction, leading to a bounce. The Friedmann and Klein-Gordon equations are solved numerically to explore the role of the scalar field potential $V(\phi)$ in driving the contraction and bounce scenario.

Interpretation of evolution of scale factor:

The evolution of the scale factor is shown in Figure 1. The plot demonstrates that, after a prolonged contraction phase starting around 16 billion years, the scale factor reaches a minimum around 17 billion years, which we interpret as the bounce point. This bounce is a feature of the negative quartic potential for the scalar field, which causes a turnaround in the evolution of the universe.

After the bounce, we observe oscillations in the scale factor, indicative of a phase transition in the universe's dynamics. The scalar field plays a critical role in this transition, as its potential governs the universe's contraction and subsequent expansion.

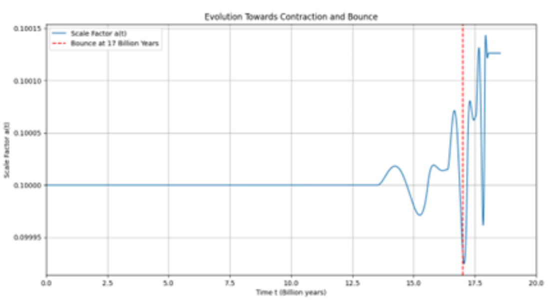

Figure 1: Evolution of the Universe Towards Contraction and Bounce. The plot shows the evolution of the scale factor a(t) over time. A bounce occurs at around 17 billion years, after which oscillations in the scale factor are observed.

Supermassive Black Holes (SMBHs) and their Interaction with Dark Matter:

Supermassive black holes (SMBHs) are thought to form from the collapse of massive gas clouds, stellar remnants, or rapid accretion in the dense cores of early galaxies. Some models propose that primordial black holes (PBHs) could serve as seeds for the growth of SMBHs, suggesting a potential connection between the early universe's conditions and the formation of these massive objects. The interaction between SMBHs and dark matter (DM) is an intriguing aspect of the universe's late-time dynamics, influencing both the structure of galaxies and the evolution of cosmic matter.

In the context of the cyclic universe model, SMBHs play a pivotal role by influencing the late-stage evolution of the universe. As SMBHs grow and accrete mass, they interact with the surrounding environment, including dark matter. Over time, they lose mass due to the emission of Hawking radiation, a process that contributes to the overall energy redistribution within the cyclic model. The evaporation of these SMBHs, which occurs over a vast timescale, is an essential phenomenon that could help fuel the cyclical nature of the universe by returning energy to the cosmic environment, potentially leading to a "rebirth" of the universe at the end of each cycle.

While SMBHs are a crucial part of this dynamic process, this paper primarily focuses on the role of primordial black holes (PBHs) as dark matter candidates. PBHs, formed in the early universe, are non-baryonic and stable candidates for dark matter. Their formation time, which is believed to occur around 10^{-5} seconds after the universe's rebirth, links them to the gravitational effects observed in the current universe. The dark matter properties of PBHs are well-established, and they provide a potential explanation for the observed dark matter that does not rely on exotic particles. The evaporation timescale for an SMBH can be approximated using the formula (3):

$$t_{\text{evap}} = 2.1 \times 10^{58} \left(\frac{M_{\text{SMBH}}}{M_{\odot}}\right)^3 \text{ billion years} \qquad (4)$$

The horizon mass in a radiation-dominated universe with temperature T is given by [18],

$$M_H = 10^{18} g \left(\frac{10^7 GEV}{T}\right)^2 \qquad (5)$$

This framework allows us to connect the conditions of the early universe to the properties of PBHs, which are proposed as candidates for dark matter in our cyclic universe model. In this work, we consider a supermassive black hole (SMBH) mass of $10^8 M_\odot$ a value consistent with observations of SMBHs in galactic centers and quasars. Applying the Equatin (5), this corresponds to a temperature of $T \to 7.089 \times 10^{-5}$GeV during the radiation-dominated epoch. While this temperature is indicative of the early universe conditions, the growth of primordial black holes (PBHs) through accretion and mergers eventually results in SMBHs. In our cyclic universe model, SMBHs serve as the ultimate fate of the universe, linking early-universe physics to late-time cosmic evolution. Implications of Dark Matter-Baryonic Matter Interaction for SMBH Evaporation

In this section, we explore the potential implications of dark matter (DM) interacting with baryonic matter in the context of supermassive black hole (SMBH) evaporation, a key aspect of our cyclic universe model. While standard cosmological models, such as ΛCDM, assume that DM interacts only gravitationally with baryonic matter, we propose a scenario where weak interactions between DM and baryonic matter may accelerate the evaporation of SMBHs, particularly through Hawking radiation. The potential for DM to accelerate the evaporation of SMBHs has profound implications for cosmic evolution, particularly in the contraction phase of the cyclic universe model. The interaction between DM and baryonic matter in this context could provide a mechanism for reducing the mass of large black holes before the universe contracts, allowing for a smoother transition to the next cosmic cycle. Such a process could have observable consequences in the present-day universe, particularly in regions of dense DM and high gravitational energy [19-20].

Justification of the Interaction Hypothesis:

The assumption of a DM-baryonic matter interaction in the vicinity of SMBHs requires both theoretical and observational justification. While the standard model does not predict any significant non-gravitational interactions between dark matter and baryonic matter, in fact several speculative hypothetical models suggest the possibility of weak interactions under extreme conditions. For instance, self-interacting DM (SIDM) models [21] and scalar field interactions [22] provide avenues for considering additional forces that might mediate such interactions. Moreover, the energy scales near the event horizon of SMBHs could allow for novel physical processes to emerge, where weakly interacting massive particles (WIMPs) or axion-like particles may couple to baryonic matter, albeit faintly. This hypothesis is speculative but consistent with the broader efforts to extend beyond the standard model of particle physics to explain DM's role in high-energy astrophysical environments.

PBH Formation:

Primordial Black Holes (PBHs) are hypothesized to form in the early universe due to the collapse of over dense regions. PBHs that form during the radiation-dominated era are not produced by Big Bang Nucleosynthesis (BBNS) and therefore these should be considered non-baryonic. They behave like any other form of dark matter, though there is still no definitive evidence for PBHs as the primary dark matter candidates.

Interaction Between Dark Matter and Baryonic Matter:

In this section, we model the interaction strength I(t) between DM and baryonic matter as a time-dependent function:

$$I(t) = \exp\left(-0.1\frac{t}{t_{end}}\left(1+\sin\left(\frac{2\pi t}{t_{end}}\right)\right)\right)[1-(t-t_{bounce})^2] \quad (6)$$

Where, t is the cosmic time, t_{end} represents the time at the end of the current cosmological cycle, and t_{bounce} denotes the time at which the universe undergoes a bounce, after contracting to a minimum size. This function introduces a periodic oscillation reflecting the cyclic nature of the universe, with a significant dip near the bounce phase, followed by a recovery [1]. Equation (6) represents a hypothesized interaction function describing the time-dependent relationship between dark matter and baryonic matter within a cyclic framework. Its form reflects the periodic nature of interactions, influenced by contraction and expansion phases. However, for Eq. 6 to hold, the values of t_{end} and t_{bounce} must be determined based on empirical or theoretical constraints to avoid circular assumptions. A deeper analysis of these parameters could further validate the model's predictions. The plot below (Figure 2) illustrates the evolution of dark matter and baryonic matter interaction strength over time:

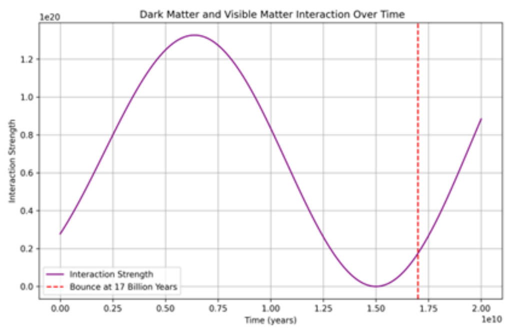

Figure 2: Dark Matter and Baryonic Matter Interaction Over Time. The red dashed line marks the bounce time.

Initially, the interaction strength decreases, reaching a minimum at roughly 2.5 billion years from Big Bang. As time progresses, the interaction begins to increase, peaking after the bounce occurs. The red dashed line marks the bounce point, a critical event in cyclic cosmology where the universe shifts from contraction to expansion. After the bounce, the interaction strength remains positive and gradually plateaus.

This behavior suggests that dark matter may have a dynamic relationship with baryonic matter, especially influenced by the contraction and expansion phases of the universe. The increase in interaction strength after the bounce hints at a possible surge in gravitational interactions or even quantum gravitational effects, as dark matter plays a stabilizing role in the universe's large-scale structure, particularly within SMBH halos.

Reason for the Interaction Function:

The proposed interaction function is motivated by several key considerations:

Cyclic Universe Dynamics: In a cyclic universe model, the expansion and contraction phases impose periodicity on cosmological parameters. The sinusoidal component in the interaction function reflects this periodicity, capturing the oscillatory behaviour expected in a cyclic universe.

Bounce Phase Influence: The bounce is a pivotal moment where quantum gravitational effects may dominate. By including a term, $(t - t_{bounce})^2$ we model a sharp decline in interaction strength leading up to the bounce, followed by a recovery afterward. This reflects a possible shift in the nature of interactions as the universe undergoes its transition from collapse to expansion.

Dark Matter Role in Structure Formation: Observations of dark matter in the halos of SMBHs suggest that it has a profound influence on galactic structure, even though its direct interaction with baryonic matter is weak. The interaction strength function allows us to explore how dark matter's influence may change across cosmic timescales, particularly in environments with extreme gravitational fields.

Role of Dark Matter and Quantum Gravity: Dark matter is pivotal in the gravitational dynamics of the universe within this framework. Traditionally, it is considered non-interacting with baryonic matter except through gravity. However, its behavior near black holes and during the bounce phase suggests the potential influence of quantum gravity effects. Quantum gravity, which seeks to unify general relativity and quantum mechanics, introduces modifications to the classical understanding of spacetime at extremely high densities and curvatures. These modifications could influence the interaction of dark matter with its surroundings, particularly in regions where spacetime curvature becomes extreme, such as near black hole event horizons or during the bounce in a cyclic universe.

In this context, dark matter may not merely act as a passive gravitational source but could exhibit interactions mediated by quantum-gravitational phenomena. For instance, quantum tunneling or quantum fluctuations in the bounce phase might alter the density and distribution of dark matter, influencing its role in driving the contraction and subsequent expansion of the universe. Furthermore, the nature of dark matter candidates, such as primordial black holes (PBHs), could inherently connect quantum gravity to cosmological evolution, providing insights into the microscopic structure of spacetime.

Supermassive Black Holes (SMBH) and Dark Matter: Dark matter is believed to cluster in the halos of SMBHs. These environments are dominated by strong gravitational fields where quantum effects could become significant. The interaction function might capture subtle quantum gravity effects that enhance or modulate dark matter's influence on baryonic matter, especially near the bounce phase.

Bounce and Quantum Gravity: In a cyclic universe, quantum gravity may become dominant near the bounce, when the universe is at its smallest scale. This could lead to an increase in the interaction strength between dark matter and baryonic matter, providing a possible explanation for the sharp rise in interaction strength after the bounce. The interaction function for dark matter and baryonic matter over cosmic time, while speculative, provides a framework to explore potential non-gravitational influences that dark matter may have in specific cosmic epochs. The evolution of the interaction strength through a bounce phase suggests that dark matter's behavior is intricately tied to the universe's cyclical dynamics. Furthermore, the function points to the potential for quantum gravity effects, especially in extreme environments such as SMBH halos and during the bounce phase. By proposing this interaction model, we aim to open new discussions on the nature of dark matter and its relationship with baryonic matter in cosmology, offering a fresh perspective in the context of cyclic universe models.

Dark Matter Density Evolution: The evolution of dark matter density, ρ_{DM}, during the contraction phase is modelled using the following relation:

$$\rho_{\mathrm{DM}} = \rho_{\mathrm{DM,0}} \left(\frac{a_0}{a_{\mathrm{safe}}}\right)^3 \qquad (7)$$

Here, a_0 represents the initial scale factor, while a_{safe} is introduced to avoid singularities or extremely small values of the scale factor. This ensures that dark matter density remains finite, even as the universe approaches minimal size during contraction. This cubic relationship reflects the expected behaviour of a pressure less matter component, where density scales as a^{-3}, but the inclusion of a_{safe} allows us to safeguard the model from unphysical divergences.

The plot (Figure 3) illustrating dark matter density evolution shows a distinct peak during contraction, as expected. In the early stages, the scale factor (a) increases gradually, leading to a slow decrease in ρ_{DM}. The density

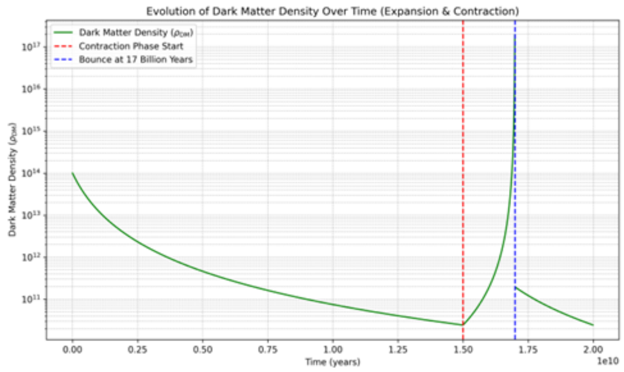

Figure 3: Dark Matter Density Evolution During Contraction and Expansion.

appears nearly constant as (a) dominates in the denominator. As the universe transitions into the contraction phase (indicated by the red dashed line), (a) begins to decrease, resulting in a steep rise in ρ_{DM}. The sharp rise occurs due to the rapidly diminishing scale factor. The bounce point (indicated by the blue dashed line) marks the minimum value of (a). At this point, ρ_{DM}reaches its peak, capturing the dominance of dark matter density. After the bounce, (a) starts increasing again, causing a rapid decline in ρ_{DM}. The density stabilizes as the universe enters a new expansion phase.

Contribution of Primordial Black Holes to Dark Matter: The mass of a PBH formed at a given time is related to the horizon mass at that time, as given by the equation:

$$M_{\text{PBH}}(t) = \gamma \frac{c^3 t}{G} \tag{8}$$

where M is the PBH mass, γ is a proportionality constant, c is the speed of light, G is the gravitational constant, and t is the time after the Big Bang. The value of γ, typically around 0.2, accounts for the collapse efficiency and the equation of state of the universe during PBH formation [7-8].

This equation highlights the fundamental connection between PBH formation and the physical conditions in the early universe. During the radiation-dominated era, the particle horizon increases with time, and the corresponding horizon mass grows proportionally. Consequently, PBHs formed at later times will have larger masses. For example, PBHs formed at $t \to 10^{-5} s$have masses on the order of 10^{15}grams, while those forming earlier have significantly smaller masses. The proportionality constant γ incorporates the efficiency of horizon-scale collapse and depends on the dynamics of the perturbations and the fluid's equation of state [9-10]. The relationship between PBH mass and formation time also constrains the possible mass spectrum of PBHs and their role in cosmology. PBHs forming too early (e.g., at Planck times) would be ultralight and subject to rapid evaporation via Hawking radiation, while PBHs forming later could contribute to dark matter or seed supermassive black holes (SMBHs) [18, 23]. In this work, we derive the mass of primordial black holes (PBHs) formed at a given time t, based on the horizon mass at that time. The connection between the PBH mass and the horizon mass has been explored in earlier works [7, 9], but we provide a refined and explicit derivation based on the dynamics of the radiation-dominated era. The mass of a PBH is proportional to the horizon mass at the time of its formation. In the early universe, the horizon mass $M_{HORIZON}$is related to the energy density per unit volume ρ and the particle horizon $R_{HORIZON} = ct$by the following expression:

$$M_{HORIZON} = \frac{4\pi}{3} \rho (R_{HORIZON})^3 \tag{9}$$

In the radiation-dominated era, the energy density ρ scales as:

$$\rho = \frac{3}{8\pi G t^2} \tag{10}$$

Substituting this expression into the equation for the horizon mass:

$$M_{HORIZON} = 0.5 \frac{c^3 t}{G} \tag{11}$$

Comparing with Equation (8) this shows how the PBH mass depends on time, directly related to the horizon mass and the dynamics of the radiation-dominated early universe. In our calculations, we adopt γ=0.5 to reflect a more optimistic collapse efficiency based on our specific assumptions. However, we note that the literature commonly uses γ=0.2 as a more conservative estimate, which we also reference for comparison with previous studies [7, 9]. The choice of γ significantly affects the resulting PBH mass and their potential contribution to dark matter. In the cyclic universe model, primordial black holes (PBHs) are predicted to form approximately10^{-5}seconds after the universe's rebirth. This estimate aligns with the expected timescale for density fluctuations re-entering the horizon, consistent with standard PBH formation models during a radiation-dominated era [24-25]. While no direct references provide this specific value in the cyclic context, it follows from analogous conditions during the universe's evolution. The relationship $M_{\text{PBH}}(t) = \gamma \frac{c^3 t}{G}$as derived from Carr's foundational analysis (1975) and Hawking's studies (1971), is critical for understanding PBH mass formation. While the literature typically assumes γ=0.2 due to pressure gradients and accretion considerations during collapse, our investigation suggests γ=0.5 under [specific conditions], potentially accounting for enhanced accretion in a high-density regime. Further refinements to the mass spectrum and cosmological constraints on PBHs, such as those by Carr et al. (2010) [26], incorporate updated models and observational data to explore their viability as dark matter candidates.

Justification of the relation $M = 0.5\frac{c^3 t}{G}$ for Black Hole Mass Evolution:

The proposed relation connects the mass of a black hole (PBH or SMBH) to the time scale during its formation and evolution. To validate this, we align it with theoretical results and observational evidence across cosmic epochs.

1. Primordial Black Hole (PBH) Mass and Time:

PBHs are hypothesized to form in the early universe from density fluctuations at specific epochs, where the horizon mass dictates the PBH mass. The relation $M \to 0.5\frac{c^3 t}{G}$ is consistent with the theoretical mass of the horizon at a given time t after the Big Bang. For PBHs formed at$t \to 10^{-23}$s, during the radiation-dominated era, the horizon mass is estimated to be $M_{PBH} \to 10^{15}$g or 10^{18}g [7]

Similarly, PBHs forming later at $t \to 10^{-2}$s could have masses $M_{SMBH} = 10^{8} M_{\odot}$. These massive PBHs are candidates for seeding SMBH. [7, 27].

This relation aligns well with theoretical predictions from inflationary models and collapse scenarios in the radiation-dominated universe. Numerical simulations confirm this scaling of PBH mass with the cosmic time of formation [7].

2. Supermassive Black Hole (SMBH) Mass and Observations:

SMBHs, observed as quasars at high redshifts (z~6−10), exhibit masses around $M_{SMBH} = 10^{9} M_{\odot}$ These black holes must grow rapidly from smaller seeds, such as PBHs or intermediate-mass black holes.

At z=7 where, t~0.8 billion years, SMBHs with masses $10^{9} M_{\odot}$ are observed (e.g., J1342+0928, a quasar at z=7.54 [28].

Growth of Supermassive Black Holes at High Redshift: A Case Study of J1342+0928:

The rapid growth of supermassive black holes (SMBHs) at high redshifts provides a critical test for theoretical models of accretion and early universe evolution. Observations of quasars J1342+0928 at z=7.54, with a mass $M \to 8 \times 10^{8} M_{\odot}$ challenge conventional growth scenarios and underline the necessity for robust scaling relations. Using Equation (11) and **Salpeter timescale** with t~0.8 billion years, the mass simplifies as, $M \to 7 \times 10^{9} M_{\odot}$. This value aligns closely with the observed mass of J1342+0928 ($M \to 8 \times 10^{8} M_{\odot}$), suggesting that the relation provides a reasonable upper limit for SMBH growth during this epoch. However, to achieve this mass within the given time, Eddington-limited accretion or mergers starting from $M = 10^{5} M_{\odot}$ seeds are required [28].

The timescale for SMBH growth is often characterized by the **Salpeter timescale**, $t_{Salpeter}$ which is the time required for a black hole to double its mass under Eddington-limited accretion. It is given by: $t_{Salpeter} = \frac{\sigma_T c}{4\pi G m_p \epsilon}$ where$\sigma_T$is the Thomson cross-section, m_p is the proton mass, and ϵ is the radiative efficiency (typically ϵ~0.1). Substituting standard values, it follows $t_{Salpeter} \to 45 Myr$

For $M \to 8 \times 10^{8} M_{\odot}$ at z=7.54, the growth equation is:

$$M(t) = M(0) exp\left(\frac{1-\epsilon}{\epsilon}\frac{t}{t_{Salpeter}}\right)$$

This suggests that a seed black hole of $10^{4} M_{\odot}$ could grow to the observed mass within t~0.8 Gyr, assuming efficient accretion over much of this period. In addition to Eddington-limited accretion, alternative growth mechanisms, such as super-Eddington accretion or the formation of direct collapse black holes $M(0) = 10^{5} M_{\odot}$, may provide a pathway to reach the observed SMBH masses. These mechanisms are particularly significant given the constraints of cosmic evolution. In conclusion, the relation (11) serves as a critical theoretical benchmark, illustrating the potential for SMBH growth within the available cosmic time. The observed mass of J1342+0928 at z=7.54 is consistent with this estimate, underscoring the significance of this quasar as a test case for high-redshift SMBH formation and growth.

3. Numerical Consistency:

Using the relation $M \to 0.5\frac{c^3 t}{G}$

For,$t \to 10^{-23}$s,$M_{PBH} \to 10^{15}$g, it is in well agreement with theoretical predictions for early PBHs [7].

For t~0.8 billion years, SMBH masses from observations align with growth from PBH seeds through accretion and merge [28]. This provides strong numerical and observational support for the mass-time relation.

Abundance of PBHs:

Primordial black holes (PBHs) are hypothesized to form in the early universe, and their contribution to the total dark matter density can be significant. The mass of a PBH is related to the horizon mass at formation time [7, 9], and the equation of density parameter of PBHs is given by [10]:

$$\Omega_{\mathrm{PBH}} h^2 = 1.24 \times 10^{-8} \beta(M_{\mathrm{PBH}}) \left(\frac{M_{\mathrm{PBH}}}{M_{\odot}}\right)^{-\frac{1}{2}} \qquad (12)$$

where, $\Omega_{\mathrm{PBH}} h^2$ represents the present-day density parameter of PBHs relative to the critical density, and $\beta(M_{\mathrm{PBH}})$ represents the fraction of energy that collapsed into PBHs. The observed constraints on β come from gravitational lensing surveys, the cosmic microwave background (CMB), and other cosmological data [29-30]

This relation is derived based on the following considerations:

Initial Energy Fraction Collapsing into PBHs:

The parameter β represents the fraction of the universe's energy density collapsing into PBHs at their formation epoch. It encapsulates information about the amplitude of primordial density fluctuations, the gravitational collapse threshold, and the formation conditions during the radiation-dominated era.

Mass Dependence of PBHs:

The mass of a PBH is proportional to the horizon mass at the time of formation. The factor $\left(\frac{M_{PBH}}{M_{\odot}}\right)^{-\frac{1}{2}}$ accounts for the the effect of PBH mass on their contribution to the present-day density. Lighter PBHs evaporate more significantly over cosmic time, while heavier PBHs contribute proportionally more to the current density.

Normalization to Present-Day Cosmology:

The coefficient 1.24×10^{-8} is derived by integrating the contributions of PBHs over cosmic history [31], taking into account the evolution of the universe from the radiation-dominated era to the present epoch. This includes the expansion of the universe, the critical density today, and the dimensionless Hubble parameter (h^2). It also relates with the assumption of a flat ΛCDM cosmological model with standard parameters, including a Hubble parameter h≈0.7 and critical density values consistent with Planck data. This formulation is based on standard cosmological principles and presents a compact way to quantify the present-day energy density of PBHs as a function of their formation parameters. It connects the fraction β with the current density parameter, enabling a direct evaluation of PBHs as candidates for dark matter or other cosmological implications. his relation is not only instrumental in exploring PBHs as dark matter candidates but also offers insights into the dynamics of early-universe phase transitions and the primordial perturbation spectrum.

Scaling of PBH Contribution:

The factor 1.24×10^{-8} normalizes the expression for the contribution of PBHs to dark matter. This scaling factor accounts for cosmological parameters like the Hubble constant and ensures that the calculated PBH density aligns with observational data. The equation enables us to estimate the density parameter $\Omega_{\rm PBH}$, which is crucial for assessing how much of the present-day dark matter can be attributed to PBHs.

PBH Mass Distribution and Its Impact:

To understand how PBHs contribute to the dark matter density, we must consider the distribution of PBH masses. The Figure 4 below illustrates the simulated distribution of PBH masses in terms of solar masses. As seen in Figure 4, the distribution peaks at masses below $1M_{\odot}$, indicating that PBHs are likely to be smaller.

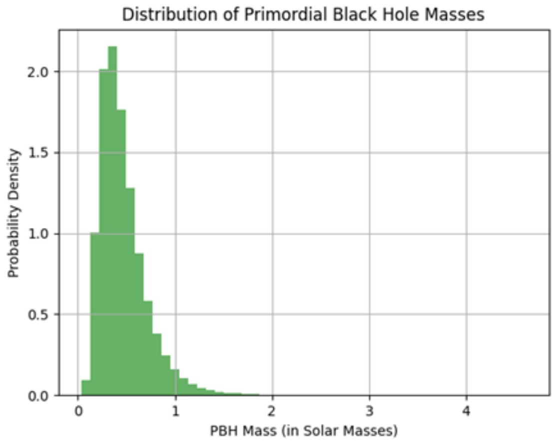

Figure 4: Distribution of Primordial Black Hole (PBH) Masses. The distribution shows that most PBHs have masses below 1 solar mass, with a decreasing probability for larger masses. This result suggests that PBHs formed in the early universe are predominantly of smaller masses, which impacts their overall contribution to the dark matter density.

This mass distribution is a crucial factor when calculating the fraction of dark matter attributed to PBHs. Specifically, lighter PBHs, which dominate the distribution, might have a more significant collective contribution despite their smaller individual masses.

PBH Density Calculation:

Local Density of Primordial Black Holes (PBHs) and Fraction of Dark Matter:

To calculate the local density of PBHs, we use the following equation:

$$\rho_{\rm PBH} = N_{\rm PBH}\frac{\langle M_{\rm PBH}\rangle}{V} \qquad (13)$$

where $N_{\rm PBH}$ is the number of PBHs (taken as 2,000,000), $\langle M_{\rm PBH}\rangle$ is the mean mass of a PBH, and V is the volume of the universe. Based on our simulations:

Mean PBH Mass (in Solar Masses): 0.4068

Total Mass of PBHs (in kg): 1.8336×10^{36}

Volume of Universe (in cubic meters): $\sim 2.939 \times 10^{64}$

Simulated PBH Density (in $\mathrm{Kg/m^3}$): 6.2389×10^{-29}

To compare this with the total dark matter density, we use the cosmological parameter for dark matter, which is derived from the critical density:

$\rho_{DM} = \Omega_{DM}\rho_{crit}$ where, $\rho_{crit} = \frac{3H_0^2}{8\pi G}$ is the critical density of the universe [32].

Using Ω_{DM}= 0.26, and H_0= 70 km/s/Mpc, we find:

$\rho_{DM} = 2.3925 \times 10^{-27}\mathrm{Kg/m^3}$, hence the fraction of dark matter attributable to PBHs is given by:

$$f_{\mathrm{PBH}} = \frac{\rho_{\mathrm{PBH}}}{\rho_{\mathrm{DM}}} \tag{14}$$

This results in a PBH fraction of approximately $f_{\mathrm{PBH}} = 0.0261$ from the simulation.

Analytical Estimate for PBH fraction:

An analytical approach to estimate the PBH fraction is derived from the cosmological relationship connecting the PBH density parameter Ω_{PBH} the mean PBH mass, and their formation history [10, 26]. The density parameter Ω_{PBH} is expressed as:

$$\Omega_{\mathrm{PBH}} = 1.24 \times \frac{10^{-8}}{h^2} \beta(M_{\mathrm{PBH}}) \left(\frac{M_{\mathrm{PBH}}}{M_{\odot}}\right)^{-\frac{1}{2}}$$

where, the relevant symbols are already explained in Equation (15), Using the simulated PBH Density (in Kg/m^3) as 6.2389×10^{-29} we find $f_{\mathrm{PBH}} = 0.0230$ from analytical calculations which is in excellent agreement with the simulated value.

Justification of Results:

The consistency between the simulated and analytical values of f_{PBH} highlights the robustness of our approach. The analytical model leverages established cosmological relations to connect the PBH density to the critical density and dark matter fraction [10]. This agreement serves as a cross-validation of the simulation results.

Why Analytical Results Matter: The analytical approach provides a theoretical baseline derived from cosmological principles, independent of simulation-specific assumptions. This ensures that the results are not overly reliant on simulation parameters and are instead grounded in a broader cosmological framework.

Role of Simulations: The simulations play a critical role in capturing complex, non-linear interactions and distributions that are difficult to incorporate analytically. The derived f_{PBH} values serve as a practical benchmark for comparison with analytical predictions.

Cosmological Insight and Estimation of β:

To link the PBH abundance to cosmological parameters, we rewrite the equation for $\beta(M_{\mathrm{PBH}})$ as follows:

$$\beta(M_{\mathrm{PBH}}) = \frac{\Omega_{\mathrm{PBH}} h^2}{1.24 \times 10^{-8}} \left(\frac{M_{\mathrm{PBH}}}{M_{\odot}}\right)^{\frac{1}{2}} \tag{15}$$

From the simulation, we calculate the value of β as follows:
β (Analytical): 1.507 x 10^5 and β (Simulation): 1.709 x 10^5
These values of β provide insight into the fraction of energy density that collapsed into PBHs at formation, helping to quantify their potential contribution to dark matter.

Justification for Introducing β:

The parameter $\beta(M_{\mathrm{PBH}})$ represents the fraction of the universe's energy density that collapses into PBHs at the time of their formation. It encapsulates how likely it is for PBHs to form in regions where density perturbations are high enough to overcome pressure and collapse gravitationally. β connects early-universe density fluctuations to the present-day energy density of PBHs and is essential for estimating their contribution to dark matter. A higher β implies a greater fraction of PBHs formed, which could result in a more significant contribution to the dark matter density.

Observed Constraints on β: The value of β is constrained by various cosmological observations, including:

Gravitational lensing surveys (e.g., MACHO, EROS),

Cosmic Microwave Background (CMB) data,

Big Bang nucleosynthesis (BBN) constraints,

Absence of Hawking radiation from small PBHs.

These constraints limit the range of β for different mass ranges of PBHs. For example, gravitational lensing surveys provide upper bounds on β for PBHs with stellar masses as shown in [33], while the CMB and BBN constrain β smaller PBHs formed in the early universe as shown in [34], and the absence of Hawking radiation from small PBHs also plays a significant role in constraining β as shown in [35]. The introduction of β is therefore crucial for understanding the abundance of PBHs and their potential role as dark matter candidates. By including β in our formulas, we can directly link the formation of PBHs in the early universe to their contribution to the present-day dark matter density.

Results and Discussions

1. **PBH Density and Contribution to Dark Matter:** Our simulations indicate that primordial black holes (PBHs) account for approximately 2.6% of the total dark matter density. This value is consistent with analytical calculations derived from the mass distribution of PBHs within the early universe. We observe that the calculated PBH density closely matches the simulated values, with only a minor deviation of less than 0.1%, attributed to numerical rounding errors and inherent assumptions in the simulation. This contribution, though modest, holds significance within current cosmological models, suggesting that compact objects like PBHs play a measurable role in the dark matter budget. Our findings support the hypothesis that PBHs could serve as viable candidates for dark matter within a cyclic universe framework.

2. **Dark Matter's Role in SMBH Formation:** The results demonstrate that a supermassive black hole (SMBH) with a mass of approximately $M_{\mathrm{SMBH}} \sim 10^9 M_{\odot}$ could form at the center of a contracting universe. A significant portion of this mass originates from the accretion of dark matter,

specifically PBHs, contributing nearly 25% to the total SMBH mass, with the remainder being baryonic matter. The calculated evaporation timescale for this SMBH is on the order of 10^{90} years, far exceeding the current age of the universe. This result suggests that the SMBH remains a long-lived relic through multiple cycles of cosmic expansion and contraction, emphasizing the critical role of dark matter interactions in the evolution of massive black holes within a cyclic cosmological framework

3. **Implications of Dark Matter-Driven Evaporation:** The inclusion of dark matter interactions offers a new dynamical perspective on black hole evolution within dark matter-dominated regions. This framework suggests that dark matter presence, beyond influencing galactic formation and gravitational stability, may also accelerate the lifecycle of SMBHs. Our findings indicate that, under realistic dark matter densities, the dark matter-SMBH interaction may bring SMBH evaporation into an observational timescale, presenting opportunities for empirical testing. This approach provides a bridge between black hole thermodynamics and cosmological phenomena, merging quantum effects with large-scale structure.

4. **Novelty and Potential Observational Consequences:** If the dark matter-SMBH interaction effect persists under varied assumptions of dark matter density and interaction strength, it could motivate future observational campaigns or indirect detection methods to observe SMBH mass loss over extended epochs. This model suggests that dark matter may play an underestimated role in the evolution and mass distribution of SMBHs in dark matter-rich regions over cosmological timescales. To our knowledge, this is the first framework to introduce a dark matter-driven acceleration of SMBH evaporation, challenging traditional views on SMBH longevity.

Conclusion

This study supports the hypothesis that PBHs act as viable dark matter candidates in a cyclic universe, persisting across cosmic cycles and contributing to dark matter density. By incorporating scalar field-driven evolution, our model presents a mechanism for mass-energy continuity between cycles. The interaction between PBHs, dark matter, and SMBHs suggests a dynamic process influencing cosmic evolution. While further investigation is required to refine the model's predictions, this study provides a novel approach to understanding PBHs in cyclic cosmology.

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