A Unique Approach to Exactly Solve Optical Pulses in Nonlinear Meta-materials

L. Nanda

Panchayat College, Bargarh, Odisha, 768028, India

Abstract

Ultra-short pulse propagation in nonlinear NRM has been investigated where a wide class of solutions for bright and dark solitons has been analyzed for distinct parameter ranges. Here the pulse propagation in nonlinear meta-material has been analytically investigated by solving the nonlinear Schrödinger's equation (NLSE) in composite media expressing frequency dispersion in the dielectric permittivity (ε) and the magnetic permeability (μ). The solutions are exactly shown to be of trigonometric & localized types. The analytical and simulation-based result has been utilized to study the intensity variation in case of a nonlinear meta-material which typically behaves as a negative refractive medium (NRM), for which both ε and μ exhibit frequency dispersion and are negative in nature. The peak of the pulse-intensity curve slowly decreases as the frequency increases towards the magnetic plasma frequency. The stability of the solitonic solutions has also been established.

Keywords: Meta-materials, NRM, NLSE, SRRs, Solitons.

*** Address of correspondence**
L. Nanda
Panchayat College, Bargarh, Odisha, 768028, India

Email: nlipsa@gmail.com

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Introduction

Nonlinear wave propagation in optics has led to innumerable innovations in several fields. The study of optical solitons in meta-materials has been a new and exciting field of research which opens the door towards numerous scientific theoretical and experimental investigations, the most notable part of which is the study of surface dynamics [1] and fiber optics [2]. In recent years meta-materials [3] have been a subject of immense theoretical and practical interest in infrared and optical frequencies due to their several applications ranging from super-resolution to cloaking. These are artificially structured materials where both the electric and the magnetic responses can be obtained at any desired frequency regime. Most meta-materials exhibit a linear response. However, nonlinearity in meta-materials can be introduced with the addition of a stack of thin wires and split ring resonators inside a nonlinear dielectric. Such type of structured materials has various applications like switching material properties from negative refractive to positive refractive, negative refraction photonic crystals and so on.

For several years significant efforts have been given by several groups to solve the higher order nonlinear Schrodinger's equation (NLSE). There have been a wide class of solutions including bright and dark solitons phase locked with the sources [4] identified for distinct parameter ranges where the solutions can be periodic as well as localized. Further investigations were carried out to study nonlinear propagation of ultra-short pulses in dispersive and negative refractive index media where a wide class of solitary waves has been obtained [5]. In any homogeneous nonlinear meta-materials, the ultra-short electromagnetic pulse propagation is often explained with the help of a system of coupled NLSE which are extracted from the Maxwell's equations. In this article a unique method has been used to exactly solve the NLSE [5] and study the solutions in a medium exhibiting frequency dependent dielectric permittivity and magnetic permeability. The solutions have also been analyzed inside a dispersive negative refractive medium (NRM).

Solution of the Required Nonlinear Schrödinger's Equation

For ultra-short-wave propagation, the group velocity dispersion (GVD) is of utmost importance. Hence ignoring the second order temporal derivatives while keeping the linear derivatives up to second order, the Generalized nonlinear Schrödinger equation (NLSE) for relatively transparent materials has been written [5] as,

$$\frac{\partial E}{\partial z}=\frac{i}{2\beta n}\left(\frac{1}{v_g^2}-\alpha\gamma-\beta\frac{\varepsilon\gamma'+\mu\alpha'}{4\pi}\right)\frac{\partial^2 E}{\partial t^2}+\frac{i}{2\beta n}\left(\frac{\partial^2 E}{\partial z^2}-\frac{2}{v_g}\frac{\partial}{\partial z}\frac{\partial E}{\partial t}\right)+\frac{i\beta\mu\chi^{(3)}}{2n}|E|^2E-\frac{\gamma\chi^{(3)}+\mu\chi^{(3)}}{2n}\frac{\partial}{\partial t}(|E|^2E), \tag{1}$$

Where

$$\alpha=\frac{\partial[\varpi\varepsilon(\varpi)]}{\partial\varpi},\quad \alpha'=\frac{\partial^2[\varpi\varepsilon(\varpi)]}{\partial\varpi^2},\quad \gamma=\frac{\partial[\varpi\mu(\varpi)]}{\partial\varpi}, \text{ and } \gamma'=\frac{\partial^2[\varpi\mu(\varpi)]}{\partial\varpi^2}.$$

The Drude-Lorentz model is used to express the dispersion of the dielectric permittivity(ε) and the magnetic permeability(μ) as $\varepsilon=1-\frac{1}{\varpi^2+i\varpi\gamma_e}$, and $\mu=1-\frac{\varpi_m^2}{\varpi^2+i\varpi\gamma_m}$ respectively, where $\varpi_m=\frac{\omega_m}{\omega_p}=0.75$, $\beta=2\pi\varpi$, and $\varpi=\omega/\omega_p$. Here ω_p, and ω_m are respectively the plasma frequencies(electric and magnetic respectively) and λ_p signifies the electric plasma wavelength. The refractive index is given by $n(\varpi)=\sqrt{\varepsilon(\varpi)\mu(\varpi)}$. The medium dispersion is shown in Fig. 1(a) for $\varpi_m=0.75$. It allows propagating waves for a positive refractive material (PRM) when $\varpi>1$, for a negative refractive material (NRM) when $\varpi<\varpi_m$, and absorptive elsewhere. The group velocity is represented as $v_g=\frac{1}{n+\varpi\frac{\partial n}{\partial\varpi}}=\left(\frac{\partial k}{\partial\varpi}\right)^{-1}=\frac{2n}{\varepsilon\gamma+\mu\alpha}$ in units of c. Eq. (1) can be re-written as

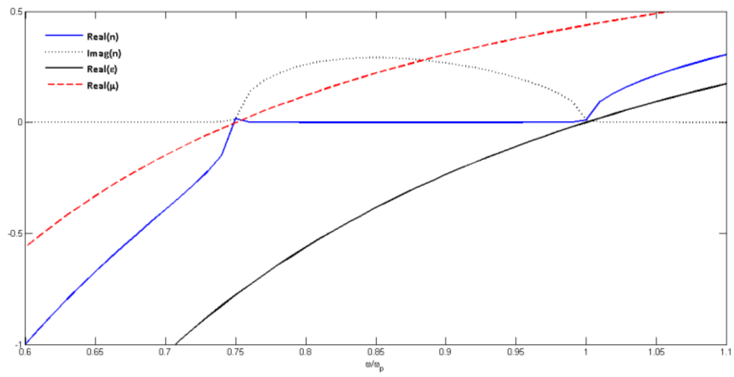
$$E_z=ia_1E_{tt}+ia_2|E|^2E-\frac{a_3}{2}\frac{\partial}{\partial t}|E|^2E+ia_5E_{zz}-ia_6E_{zt}. \tag{2}$$

Eq. (2) is further written as

$$iE_z+a_1E_{tt}+a_2|E|^2E+ia_3|E|^2E_t+ia_4E^2E_t^*+a_5E_{zz}-a_6E_{zt}=0. \tag{3}$$

Here $E(z,t)$ denotes the complex electric field with the subscripts z and t corresponding to the partial derivatives for space and time respectively.

The different coefficients in Eq. (3) are
$a_1=\frac{1}{2\beta n}\left(\frac{1}{v_g^2}-\alpha\gamma-\beta\frac{\varepsilon\gamma'+\mu\alpha'}{4\pi}\right)$,
where $-a_1$ signifies the GVD. The self-phase modulation (SPM) is given by



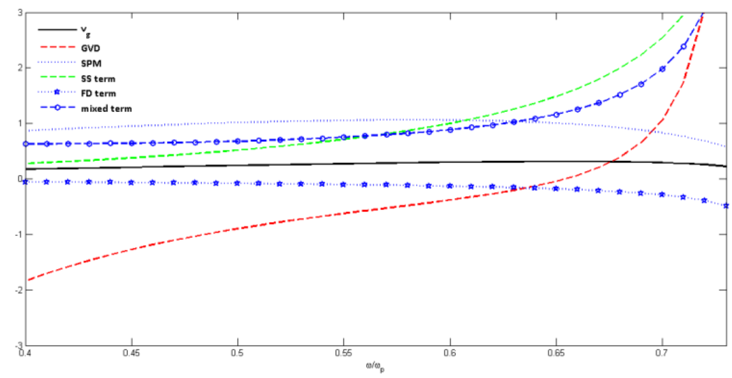


Figure 1: (a) The medium dispersion with the refractive index ($n(\varpi)$), and real parts of $\varepsilon(\varpi)$, and $\mu(\varpi)$ shown as a function of $\varpi=\omega/\omega_p$. **(b)** the different coefficients in the NLSE are shown as a function of ϖ.

$a_2 = \frac{\beta\mu\chi^{(3)}}{2n}$,

whereas the self-steepening (SS) term is represented by $-a_4$ where $a_3 = \frac{\gamma\chi^{(3)}+\mu\chi^{(3)}}{n} = 2a_4$.

The Fresnel diffraction (FD) coefficient is given by $a_5 = \frac{1}{2\beta n}$, which mostly defines the pseudo-quintic nonlinearity. Further $a_6 = \frac{1}{\beta n v_g}$, where $-a_6$ represents the mixed spatio-temporal second order term. The various coefficients of the NLSE are shown in Fig. 1(b). Here the complex form of the electric field is used which is function of the travelling coordinate $\xi = \eta(z - v_g t)$, where η signifies the width. The electric field is represented by $E(z,t) = \rho(\xi)e^{i\chi(\xi)}$, where $\rho(\xi)$ stands for the amplitude and $\chi(\xi)$ signifies the phase where both are taken as real. Substituting the electric field in Eq. (3), and writing down all the terms together, we get,

$$\eta(i\rho' - \rho\chi')e^{i\chi} + a_1\eta^2 v_g^2(\rho'' + 2i\rho'\chi' + i\chi''\rho - \chi'^2\rho)e^{i\chi} + a_2\rho^3 e^{i\chi} - i\eta v_g a_3\rho^2(\rho' + i\rho\chi')e^{i\chi} - ia_4\rho^2\eta v_g(\rho' - i\chi'\rho)e^{i\chi} + a_5\eta^2(\rho'' + 2i\rho'\chi' + i\rho\chi'' - \rho\chi'^2)e^{i\chi} + a_6\eta^2 v_g(\rho'' + 2i\rho'\chi' + i\chi''\rho - \chi'^2\rho)e^{i\chi} = 0, \tag{4}$$

where prime stands for the differentiation with respect to ξ. The R.H.S in Eq. (4) is zero means that both components of the complex expression have to separately vanish. They are represented by a pair of coupled equations,

$$-\eta\rho\chi' + a_1\eta^2 v_g^2\rho'' - a_1\eta^2 v_g^2\chi'^2\rho + a_2\rho^3 + \eta v_g a_3\rho^3\chi' - a_4\rho^3\eta v_g\chi' + a_5\eta^2\rho'' - a_5\eta^2\rho\chi'^2 + a_6\eta^2 v_g\rho'' - a_6\eta^2 v_g\chi'^2\rho = 0. \tag{5}$$

and

$$\eta\rho' + 2a_1\eta^2 v_g^2\rho'\chi' + a_1\eta^2 v_g^2\chi''\rho - \eta v_g a_3\rho^2\rho' - a_4\rho^2\eta v_g\rho' + 2a_5\eta^2\rho'\chi' + a_5\eta^2\rho\chi'' + 2a_6\eta^2 v_g\rho'\chi' + a_6\eta^2 v_g\chi''\rho = 0. \tag{6}$$

Eq. (6) after integrating once gives

$$\chi'\rho^2(a_1\eta^2 v_g^2 + a_5\eta^2 + a_6\eta^2 v_g) = k_1 - \frac{\eta\rho^2}{2} + \frac{(a_3 + a_4)\eta v_g}{4}\rho^4, \tag{7}$$

where k_1 is the constant of integration. Substituting the value of χ' from Eq. (7) in Eq. (5) and simplifying further we get,

$$\theta_1\rho'' + \theta_2\rho + \theta_3\rho^3 + \theta_4\rho^5 = \frac{\theta_5}{\rho^3}, \tag{8}$$

Where $\theta_1 = \eta^2\theta$ with $\theta = a_1 v_g^2 + a_5 + a_6 v_g$,

$\theta_2 = \frac{(-a_3\eta v_g k_1 + \eta^2)}{4\eta^2\theta}$, $\theta_3 = a_2 - \frac{a_3 v_g}{4\theta}$, $\theta_4 = \frac{3a_3{}^2 v_g^2}{64\theta}$,

$\theta_5 = \frac{k_1^2}{\eta^2\theta}$.

After integrating Eq. (8) further, the conserved value of k_2 is obtained which is a constant of integration and is given as,

$$k_2 = \theta_1\rho'^2 + \theta_2\rho^2 + \frac{\theta_3}{2}\rho^4 + \frac{\theta_4}{3}\rho^6 + \frac{\theta_5}{\rho^2}. \tag{9}$$

Here $\rho = \sqrt{f + h}$, where $f(\xi)$ is taken as a real function with h as a constant parameter. Taking $(f + h) > 0$, Eq. (9) is expressed in terms of f, which after successive differentiation gives:

$$2\theta_1 f'' + (8\theta_2 + 12\theta_3 h + 16\theta_4 h^2)f + (6\theta_3 + 16\theta_4 h)f^2 + \frac{16\theta_4}{3}f^3 + (8h\theta_2 + 6h^2\theta_3 + \frac{16h^3\theta_4}{3} - 4k_2) = 0. \tag{10}$$

By making the coefficient of f^2 vanish, the above equation converges into a cubic equation which after simplifying gives $6\theta_3 + 16\theta_4 h = 0, \Rightarrow h = -\frac{3\theta_3}{8\theta_4}$.

$$\eta^2 f'' + c_1 f + c_2 f^3 + c_3 = 0, \tag{11}$$

where $c_1 = \frac{8\theta_2 + 12\theta_3 h + 16\theta_4 h^2}{2\theta}$,

$c_3 = \frac{(8h\theta_2 + 6h^2\theta_3 + \frac{16h^3\theta_4}{3} - 4k_2)}{2\theta}$. and $c_2 = \frac{8\theta_4}{3\theta}$,

The solution of Eq. (11) is mostly of rational type which is given as $f = \frac{A + Bf1^2}{1 + Df1^2}$, where there are various possible solutions of $f1$, for example, $f1 = C_n(\xi, m)$, with m taken as the modulus parameter. Substituting the expression of f in the cubic equation given by Eq. (11), a polynomial of $C_n(\xi, m)$, is obtained. The consistency conditions can be extracted by putting the coefficients for all terms individually equal to zero. In this article the solution for $m = 0$ is shown where, $C_n(\xi, m = 0) = Cos(\xi)$. Substituting $A = 0, B \neq 0$ (which are the allowed values) in the consistency conditions, the coefficients obtained are $B = -\frac{C_3}{2\eta^2}$, $D = -\frac{2}{3}$, and $\eta^2 = -\frac{C_1}{4}$, with the consistency condition $c_1 = \left(-\frac{27C_2C_3^2}{2}\right)^{\frac{1}{3}}$, which implies that c_1 is negative. The obtained solutions of Eq. (11) are periodic and are represented by,

$$f(\xi) = 2\left(\frac{c_3}{c_1}\right)\frac{\cos^2(\xi)}{1-\frac{2}{3}\cos^2(\xi)} \tag{12}$$

Solution in a Meta-material

In the above analysis, the analytical and simulation-based solution of an optical pulse traversing through a nonlinear dispersive medium has been obtained. The results have been analyzed for the case of artificially structured meta-materials which exhibit material dispersion. Further the optical wave propagation has been investigated in the nonlinear NRM which can be achieved within a particular frequency range (Fig.1). The pulse-intensity has been plotted with respect to the scaled frequency ($\varpi = \omega/\omega_p$) and (ξ) (Fig.2). It is apparent that the peaks corresponding to the pulse-intensity curve decreases with the increase in frequency towards ϖ_m .

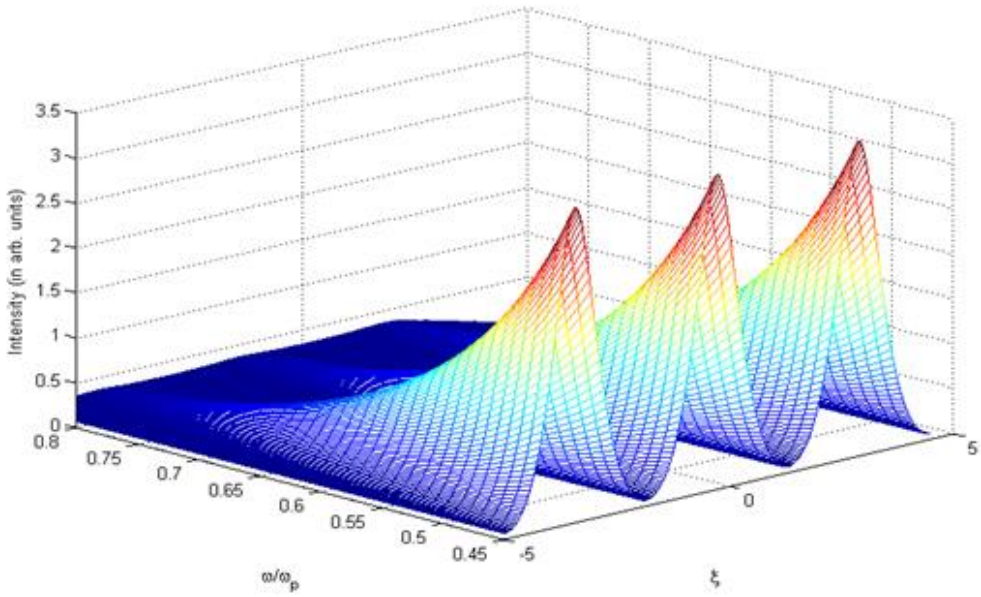


Figure 2: Pulse-intensity plotted as a function of (ξ) and the scaled frequency ($\varpi = \omega/\omega_p$) in the negative refractive medium (NRM).

Conclusions

A unique method has been used to obtain an exact analytical and simulation-based solution of an optical pulse traversing through a nonlinear medium having frequency dispersion. It has been shown that the intensity is dependent on frequency when the medium is taken to be dispersive. The solutions have been investigated in an NRM which exhibits dispersive dispersion. The solutions have been obtained within a particular frequency range. It has been observed that the pulse-intensity decreases with the increase in frequency towards ϖ_m . The stability of the solution is also well established.

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