Josephson current across a Double Quantum Dot Josephson junction in T-Shape Configuration

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Abstract

By implementing the Keldysh non-equilibrium Green's function equation of motion approach, Josephson current has been examined across a T-shaped uncorrelated double quantum dot Josephson junction. The behavior of the Josephson current as a function of the main quantum dot energy level for varied interdot tunneling and different dot-lead coupling strengths is examined. With this configuration, we illustrate that the side-attached quantum dot offers an alternative route for electron transmission, which modifies the Josephson current by varying interdot tunneling. Further, we also investigate how the dot-lead coupling strengths affect the Josephson current.

Keywords: Josephson Current, Double Quantum Dots, Equation of Motion.

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Introduction

Josephson transport through quantum dot junctions has been widely implemented in nanoelectronic devices such as superconducting quantum interference devices (SQUIDS) [1], Cooper pair splitters [2], and superconducting quantum bits [3]. The development of nanofabrication techniques enabled the creation of devices with superconducting leads coupled to quantum dots (QDs). QDs are nanoscopic semiconductor structures that confine electrons to zero dimensions. These QDs, like atoms, have a distinct energy level due to quantum confinement [4]. In the superconductor-quantum dot-superconductor system, Josephson current flows as a result of the creation of subgap states, also known as Andreev Bound states (ABS). The Andreev reflection process happens at the interfaces between the quantum dot and the superconducting leads in a superconductor-quantum dot system. When an electron from one of the leads meets the dot, it can be retro-reflected as a hole, resulting in the creation of a Cooper pair in the superconducting lead. Numerous theoretical [5-9] and experimental [10-12] studies of charge transport in superconductor-quantum dot-superconductor systems have been addressed in recent years.

Additionally, the charge transport properties of Josephson junctions with double quantum dots have been explored in various research papers. These junctions are characterized by the coupling of two quantum dots in a series, parallel, or T-shape arrangement with superconducting leads [13-18]. Because of adjustable characteristics such as quantum dot energy levels, dot-lead coupling strength, and interdot tunneling, studying quantum electronic transport across systems where double quantum dots (DQD's) are coupled to superconducting leads has been a research area in recent years. Whether the dots are firmly or weakly linked is determined by interdot tunneling. DQD's controllable adjustable characteristics make it useful for analyzing transport phenomena such as the Coulomb blockade, the Kondo effect, quantum coherence, and so on.

This paper examines the Josephson current in a system with double quantum dots coupled to two superconducting leads in a T-shape configuration. In this setup ($S - T$ DQD $- S$) (see figure 1), the main quantum dot (QD_1) is coupled directly to the leads, whereas the side quantum dot (QD_2) is connected to the QD_1 but not to the superconducting leads. In this study, we investigated the transport properties of a

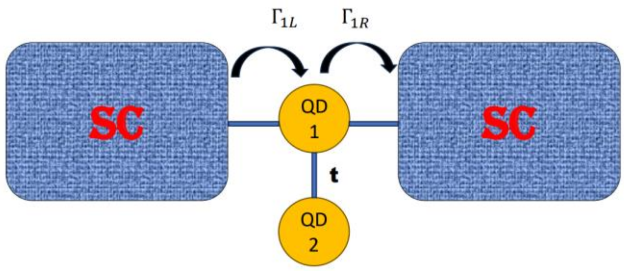

Figure 1: Schematic diagram of T-shaped double quantum dot Josephson Junction

T-shaped double quantum dot Josephson junction by using Keldysh non-equilibrium Green's equation of motion technique [19-20]. We studied the interdot tunneling and dot-lead coupling dependency of the Josephson current.

Theoretical Formulation

Generalized Anderson and BCS Hamiltonians in second quantization formalism are used to model the double quantum dots system in T-shape configuration as follows:

$$\hat{H} = \hat{H}_{leads} + \hat{H}_{dots} + \hat{H}_{interdot-tuneling} + \hat{H}_{dot-lead}. \quad (1)$$

where

$$\hat{H}_{leads} = \sum_{k\sigma,\alpha} \epsilon_{k\alpha} c^{\dagger}_{k\sigma,\alpha} c_{k\sigma,\alpha} - \left(\sum_{k\alpha} \Delta_{\alpha} c^{\dagger}_{k\uparrow,\alpha} c^{\dagger}_{-k\downarrow,\alpha} + h.c.\right)$$

$$\hat{H}_{QD} = \sum_{i=1}^{i=2} \sum_{\sigma} \epsilon_{d_{i\sigma}} d^{\dagger}_{i\sigma} d_{i\sigma}$$

$$\hat{H}_{interdot-tuneling} = \sum_{i\sigma} t d^{\dagger}_{1\sigma} d_{2\sigma} + h.c$$

$$\hat{H}_{dot-lead} = \sum_{k\sigma,\alpha} V_{k,\alpha} c^{\dagger}_{k\sigma,\alpha} d_{1\sigma} + h.c$$

$\hat{H}_{leads}$ is the Hamiltonian of BCS superconducting leads. In the first term $\varepsilon_{k\propto}$ is the energy of superconducting leads ($\alpha \in$ L, R) and $c^{\dagger}_{k\sigma,\alpha}$($c_{k\sigma,\alpha}$) is the creation (annihilation) operator of electrons with spin σ(↑,↓) and wave vector $\vec{k}$. The second term denotes the interaction between Cooper pairs where Δ_{α} is the superconducting order parameter and is given as $\Delta_{\alpha} = |\Delta_0| e^{i\varphi\alpha}$.

$\hat{H}_{dots}$ denotes the Hamiltonian of double quantum dots. The energy of main quantum dot (QD_1) and side quantum dot (QD_2) is given by ε_{di} with fermionic operators ($d^{\dagger}_{i\sigma}$ and $d_{i\sigma}$).

$\hat{H}_{interdot-tunneling}$ is the Hamiltonian for tunneling between both quantum dots and the amplitude of interdot tunneling is symbolized by t.

$\hat{H}_{dots-lead}$ represents the hamiltonian for tunnelling between QD_1 and superconducting leads where $V_{k,\alpha}$ is the tunneling strength between leads and QD_1.

To compute the spectral and transport properties of S − T DQD − S system, the single-particle retarded Green's function of main quantum dot QD1 is required. We have resolved the above-mentioned Hamiltonian (Eq. 1) using Green's equation of motion approach (EOM).

$$\langle\langle d_{\sigma}(t); d^{\dagger}_{\sigma}(0)\rangle\rangle = -i\theta(t)\langle [d_{\sigma}(t), d^{\dagger}_{\sigma}(0)]_{+}\rangle$$

The following equation of motion should be satisfied by the Fourier transform of the above retarded Green's function.

$$\omega\langle\langle d_{\sigma}|d^{\dagger}_{\sigma}\rangle\rangle_{\omega} = \langle\{d_{\sigma}, d^{\dagger}_{\sigma}\}_{+}\rangle + \langle\langle [d_{\sigma}, H]_{-}|d^{\dagger}_{\sigma}\rangle\rangle_{\omega} \quad (2)$$

The Green's function EOM technique is used to get the coupled set of equations (Eq. 2), and when the closed set of coupled equations is solved, the expression for the single particle retarded Green's function of QD_1 may be written as follows:

$$G^{r}{}_{11}(\omega) = \langle\langle d_{1\uparrow}|d^{\dagger}_{1\uparrow}\rangle\rangle = \frac{\omega + \varepsilon_{d_1} - \frac{t^2}{\omega + \varepsilon_{d_2}} - I_1}{(\omega + \varepsilon_{d_1} - \frac{t^2}{\omega + \varepsilon_{d_2}} - I_1)(\omega - \varepsilon_{d_1} - \frac{t^2}{\omega - \varepsilon_{d_2}} - I_2) - (I_3)^2} \quad (3)$$

In the Green's function shown above, I_1, I_2 and I_3 refer to the diagonal and off-diagonal parts of self-energy, respectively, which correspond to the pairing that is produced as a result of the connection between the quantum dot and superconducting leads. The expressions for I_1, I_2 and I_3 are written as follows:

$$I_1 = I_2 = -\sum_{\alpha\in L,R} \left(\frac{\Gamma_{\alpha}\omega}{\sqrt{\Delta^2_{\alpha} - \omega^2}} \theta(\Delta - |\omega|) + i(\frac{\Gamma_{\alpha}|\omega|}{\sqrt{\omega^2 - \Delta^2_{\alpha}}} \theta(|\omega| - \Delta) \right) \quad (4)$$

$$I_3 = -\sum_{\alpha\in L,R} \left(\frac{\Gamma_{\alpha}\Delta_{\alpha}}{\sqrt{\Delta^2_{\alpha} - \omega^2}} \theta(\Delta - |\omega|) + i(\frac{\Gamma_{\alpha}\Delta_{\alpha}}{\sqrt{\omega^2 - \Delta^2_{\alpha}}} \theta(|\omega| - \Delta) \right) \quad (5)$$

The expression of Josephson current can be written as [7, 21]. In these references, authors gave a full derivation for the expression of Josephson current.

$$I_J = \frac{2e}{h}\int d\omega \frac{\Gamma_\alpha^2 \Delta_\alpha^2}{\omega^2 - \Delta_\alpha^2} f(\omega) sin\phi Im\left[\frac{-1}{A(\omega)}\right] \quad (6)$$

where Γ_α is the main dot-lead coupling strength, Δ_α is the superconducting gap parameter, f(ω) is the Fermi-distribution function, and A(ω) is the denominator of the single particle retarded Green function (Eq.3).

Results And Discussion

In this section, on the basis of the theoretical formulation discussed above, we compute the Josephson current as a function of QD_1 energy level for distinct values of interdot hopping (t) and dot-lead coupling (Γ_α).

Δ_0, which is in *meV*, is taken as the energy unit. For simplicity, we assume both the superconducting leads are identical i.e. $\Delta_L = \Delta_R = \Delta$ and also considered symmetric tunneling between QD′s and superconducting leads i.e. $\Gamma_L = \Gamma_R = \Gamma$.

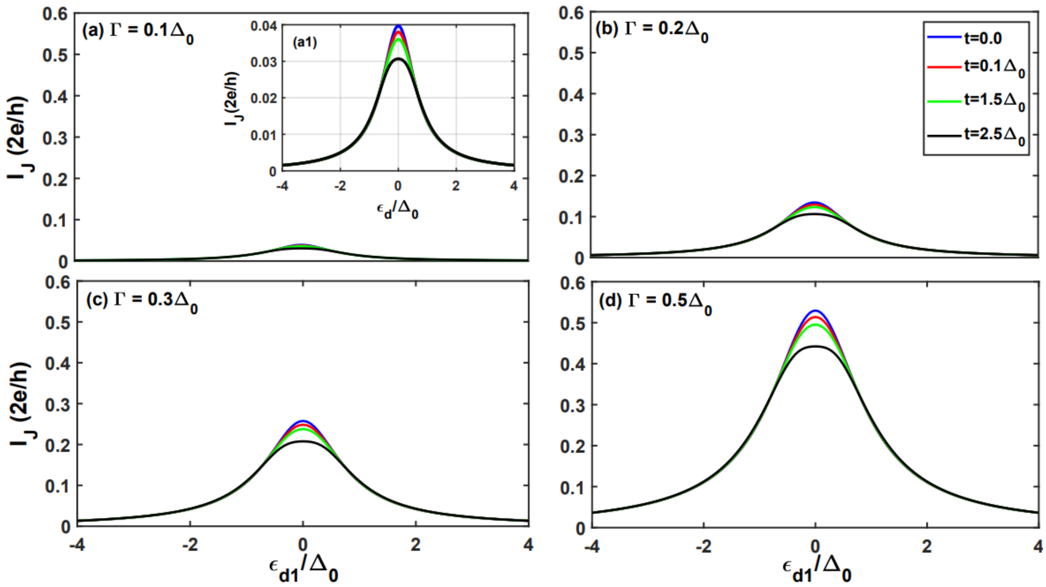

Figure 2: Josephson current (I_J) vs QD_1 energy level for different interdot hopping (t) for various dot-lead coupling strengths. The other parameters are $T = 0.2\Delta_0$, $\phi = \frac{\pi}{2}$. Inset (a1) is the magnified view of the $\Gamma = 0.1\Delta_0$ case for different interdot hopping.

In figure 2 (a-d), Josehsonn current shows a symmetric resonant peak centered at $\varepsilon_d = 0$, and the peak height suppresses with the increasing interdot hopping (t). This behavior of Josephson current can be explained by the interference of two paths. In the absence of QD_2 i.e. t=0, electrons directly transport from the one superconducting lead to QD_1 and then the other superconducting lead, inducing a larger peak (blue solid line in Figure 2). When QD_2 is connected to the QD_1 i.e. t > 0, electrons tend to tunnel into QD_2 and thus Josephson current suppresses.

Further in the same figure (a-d), we show the impact of dot-lead coupling strength Γ on Josephson current.

The enhancement in the amplitude of Josephson current with increasing Γ is obvious and shown in figure 2 (a-d) for different dot-lead coupling strengths. As only QD_1 is connected to superconducting leads so on increasing the dot-lead coupling strength the electrons tunnel directly from one superconducting lead to the other superconducting lead and thus enhances the amplitude of Josephson current.

Conclusion

We have addressed the Josephson current versus QD_1 energy level across the double quantum dot Josephson junction in a T-shape configuration. It is observed that interdot tunneling and dot-lead coupling strengths of quantum dots perform a key role in the tunability of Josephson current. It is observed that Josephson current exhibits a symmetric resonant peak centered at $\varepsilon_d = 0$, and the peak height suppresses with increasing interdot tunneling. We also show that Josephson current enhances with increasing dot-lead coupling strengths. In future superconducting devices, the double quantum dot-based Josephson junction might provide tunable supercurrents and new noise features. This research may be expanded to include multi-dot and multi-terminal Josephson junctions.

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